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A CLASS BOOK OF PHYSICS

GREGORY AND HADLEY

PARTS IV & V

LIGHT · SOUND

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A

CLASS BOOK OF PHYSICS

BY

SIR RICHARD GREGORY

AND

H. E. HADLEY, B.Sc.

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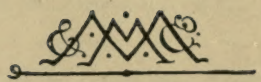

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A CLASS BOOK
OF
PHYSICS

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BY
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PARTS IV. AND V.
LIGHT, AND SOUND

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PREFACE

THERE is a diversity of opinion as to the value of text-books in the teaching of Science, and of practice as to their use. Prof. Armstrong has urged that "each child should write its own text-book and be taught to regard it as a holy possession"; and in a few schools this exalted doctrine is accepted in theory if not in fact. The principle is sound enough when applied by the private tutor (and the authors have no desire to belittle it) but it cannot be adapted satisfactorily to the work-a-day conditions of science teaching in schools where a large number of pupils of varying capacity and limited resource are receiving instruction in classes. In most cases, the time available for a science course will not permit the go-as-you-please pace postulated by some educational reformers as essential to good work, even if it be assumed that each pupil not only realises the necessity of working out his own intellectual salvation but is capable also of constructing his own road to it. How few pupils there are who possess the motive and purpose required for successful scientific study without assistance from a text-book is known only to the practical teacher.

Text-books may not be essential in the early work in science, but after pupils have acquired some familiarity with the scientific method it is desirable to fix ideas by more systematic study. Without practical experience there is no real scientific knowledge; but, on the other hand, unless laboratory work is accompanied by descriptive reading or lectures, it usually ends in a nebulous state of mind equally dispiriting to the teacher and pupil. The study of science is best encouraged by a right

combination of experiment, discussion, and reading, and the only limit to the amount of either is that of time. The exigencies of the school time-table do not permit the pursuit of an indefinite course of practical work or the leisurely consultation of authoritative treatises upon the subjects under consideration. For this reason books are demanded which present more or less concisely the essential principles of a branch of science and weave the scattered threads of thought into a fabric of definite pattern and reasonable dimensions.

It is not pretended that this volume is other than a text-book designed to facilitate the work of the teacher and concentrate the attention of the pupil: its purpose is not so much to inspire as to instruct. Between the prolix popular work, the treatise for reference, and the students' manual there are sharp lines of distinction; and though each has a place in scientific literature their respective merits must be judged by different standards. A text-book should be concise in description and precise in instructions for practical work: it should provide not only substance and guidance for study, but also exercises to test the increase of effective mental equipment and to cultivate the art of clear expression. In class teaching it is not sufficient to prescribe topics for reading or experiment: there must be some means of determining whether the work has been performed. A well kept record of laboratory experiments is doubtless an excellent index of progress made, but unless it is supplemented by exercises intended to test the ability to apply the results and conclusions arrived at to the solution of related problems, and the interpretation of wider experience, it may prove to be a vain thing. Such a record is a measure of the kinetic energy of a pupil's laboratory performances but not of the extent to which the energy has been transformed into potential power. It is good discipline and helpful teaching, therefore, by oral or written questions to make a periodic valuation of the capacity of students to comprehend the full meaning of the work done.

Considerations of time and space determine the scope of a

science course and of a text-book; and in neither case should sins of omission be judged so severely as those of commission. In most schools the chief work in science consists of the subjects of fundamental physical measurements and heat included in Parts I., II., and III. of the complete volume. Practical acquaintance with these aspects of Physics is a necessary preliminary or accompaniment to the successful study of physical or chemical science; and this must be the excuse for the apparently excessive amount of space devoted to them. Light, Sound, Magnetism and Electricity are regarded now as special subjects to be taken up selectively after a foundation has been laid in physical principles. Local circumstances and the requirements of examining bodies decide which of these subjects shall be studied, but in any case it is hoped that a satisfactory first course of systematic work will be found in the following pages.

The standard of each Part of the volume is roughly that which may be expected reasonably of pupils from about fourteen to sixteen years of age. Probably few pupils will work through the complete course, but an endeavour has been made to provide for the requirements of students who intend to present themselves as candidates in elementary examinations in any branch of physics.

By the kind consent of Mr. A. T. Simmons, parts of books of which he is joint author have been adapted for use in the present volume. Adequate thanks cannot be expressed for the self-abnegation thus manifested by the friend and long-time colleague of the authors. Most of the illustrations are new, but a few are based upon figures in other books published by Messrs. Macmillan & Co., Ltd., to whom the authors are glad to acknowledge their indebtedness.

In almost every case the exercises at the end of the chapters are from papers set at various examinations. Among the papers from which suitable questions have been selected are those of Oxford and Cambridge Locals, London University Matriculation and School examinations, Intermediate Scholarships of the

London County Council, and the Board of Education examinations. It will be noticed that these questions are frequently of the nature of problems to be solved—either practically or otherwise—and cannot be answered by the mere repetition of the substance of the text preceding them. For permission to reproduce questions from examination papers thanks are expressed gladly to the Controller of H.M. Stationery Office, the Senate of the University of London, the Delegates of the Oxford Local Examinations, the Cambridge Local Examinations and Lectures Syndicate, and the London County Council Education Committee.

R. A. GREGORY.
H. E. HADLEY.

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PART IV.

LIGHT.

CHAPTER XVIII.

PROPAGATION OF LIGHT. SHADOWS. PHOTOMETRY.

Light is a form of radiation.—In considering, in a part of the previous chapter, the ways in which heat can be transferred from one place to another, it was seen that the heat of the sun reaches the earth by radiation. These solar radiations comprise what collectively is called sunlight; they are conveyed in the form of waves through the medium ether, which is believed to pervade all space, and may be referred to conveniently as **ether-waves**. These ether-waves are of various lengths and can produce different effects. If they fall upon our bodies the longer waves may be absorbed, and the energy of the wave-motion become converted into **heat**; if they fall upon the retina of an eye, the shorter waves may produce a sensation of light, and the waves are then spoken of as **light**; falling upon a photographic plate or upon a green leaf, the shortest ether-waves may produce chemical effects, and these waves are referred to as **actinic**. But, in their passage through the ether, ether-waves do not give rise to any of these effects; they are simply waves transferring energy by wave-motion.

Opaque and transparent media.—Ether-waves are not transmitted through all media with equal facility. Substances such as iron and other metals, wood, or stone, through which the waves which produce the sensation of light cannot be transmitted, are termed **opaque**; others, such as glass, water, or air through

which such waves are transmitted readily, are termed **transparent**. A few media are known which transmit a portion only of the light falling upon their surface, and these are termed **translucent**; fog, white paper, ground glass, porcelain, and a mixture of milk and water are examples of translucent media. At the same time, the distinction between opaque and transparent media is not absolutely definite, since more or less light may be transmitted through media usually regarded as opaque, providing that a layer of sufficient thinness is used. Thus, a sheet of gold leaf, when supported between two sheets of glass, is somewhat transparent; similarly, a very thin section of a stone is transparent, and a thin shaving of wood is translucent. The fact that very little light penetrates to great depths beneath the surface of water shows that this medium is transparent only when thin layers are used.

Light travels in straight lines.—It can be shown by the observation of several simple phenomena that light travels in straight lines through any homogeneous transparent medium. Any straight line in such a medium along which light is propagated may be termed a **ray** of light. Rays of light proceed in all directions from every point of a luminous body; a small assemblage of rays included within a cone of very small angle, and proceeding at its apex from a luminous point, is termed a **diverging pencil** or **beam**; if the rays converge towards a point they constitute a **converging pencil**. When the luminous point is at an infinite distance, the rays of light are practically parallel to one another, and a small assemblage of the rays is termed a **parallel pencil** or **beam**.

That light rays travel in straight lines in ordinary circumstances can be shown at once by examining the paths of the rays as they pass through a hole in the shutter of a darkened room. Though the light-waves are not themselves visible, yet the path of the light becomes apparent, because the minute particles of dust in the air are rendered luminous by the vibrations of the ether being reflected from them. If there were no dust particles in the room the beam of light would be invisible. When the path of a beam is made visible by smoke or dust it is seen to be a straight line. That light travels in straight lines may, indeed, be inferred from several everyday experiences. We cannot see round a corner; if light travelled

through a uniform medium in lines that were sometimes bent, there is no reason why we should not. Or, again, everyone knows that it is only necessary to put a small obstacle in the path of the light from a luminous body to shut out completely our view of it.

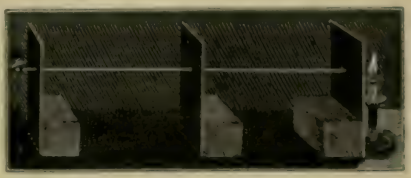

Fig. 150.—Experiment to show that light travels in straight line.

Expt. 167.— **Rectilinear propagation.** Place three cards together and pierce a small hole through them. Support the cards as in Fig. 150. The light of the candle can be seen only when the holes are in a straight line. What is true of light applies equally to all other kinds of radiation.

Inverted images produced by a pin-hole.—When an object is viewed through a pin-hole camera, it is seen to be upside down upon the screen. Similarly, all images produced by a small aperture are inverted. This inversion is a direct consequence of the fact that light travels in straight lines. That this is really the case can be understood fully by the following simple considerations.

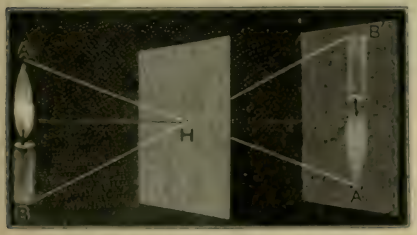

Fig. 151.—Explanation of the inversion of images seen through a pin-hole.

Let H, in Fig. 151, be the pin-hole, and AB the candle. Rays are sent out in all directions by every point of the candle, but of all the rays from one point, such as A, only that in the direction AH can pass through the hole and form an image A'. Similarly, the only ray from B which can get through the hole is BH, so an image of B is formed at B'. The same reasoning applies to any part of the candle, hence a complete inverted image is produced.

If the light which passes through a small hole in the shutter of an otherwise dark room be caught by a screen of cardboard, a coloured, inverted image of the sky and landscape will be seen.

By using a pin-hole camera, a photograph of the view can be obtained (Fig. 152). A pin-hole camera can be made from a light-proof box having a minute hole at one end, or one can be purchased for a few pence. The bright circles of light seen under trees in summer are really images of the sun formed by the small spaces between the leaves.

FIG. 152.—Photograph obtained with a pin-hole camera, by Mr. F. Butterworth.

Size of image produced by a pin-hole.—That the size of the image depends upon the distance of the screen from the pin-hole is proved practically by varying the distance of the screen from the pin-hole and measuring the length of the image. The greater the distance of the screen the longer the image. The reason for this alteration in the size of the image is a simple one. The rays of light from the top and bottom of the object travel through the pin-hole, and since one is travelling upwards and the other downwards, they will be farther apart the greater the distance they travel. Consequently, the image is longer the more the screen is moved from the pin-hole.

The relation between the sizes of the object and image according to their distances from the aperture is

$$\frac{\text{Length of object}}{\text{Length of image}} = \frac{\text{distance of object from aperture}}{\text{distance of image from aperture}}.$$

The larger the image the less bright it is, because the small amount of light which passes through the aperture is spread over a greater area in the case of the enlarged image.

Illumination due to overlapping of images.—When a pin-hole is made in the front face of a pin-hole camera, an image of the

bright object looked at is formed on the screen in the manner described in the preceding paragraphs. If a second hole be pierced, a second image is obtained. When the number of holes is increased steadily one at a time, the images, it is observed, start overlapping, at the same time becoming blurred. When the number of images has become considerable, no separate image can be distinguished; **diffused light**, as it is called, is produced, and the screen is illuminated in the ordinary way.

Intensity of light.—In proceeding from the source of illumination, light spreads out as indicated in Fig. 153, so that though each ray retains its original intensity the number of rays which illuminate a given area depends upon the distance of that area from the luminous source S. At twice the distance the rays are spread over four times the area, so their illuminating effect, as at M, is only one-fourth of what it is at *m*. The amount of light received from a luminous source is thus inversely proportional to the square of the distance from the source.

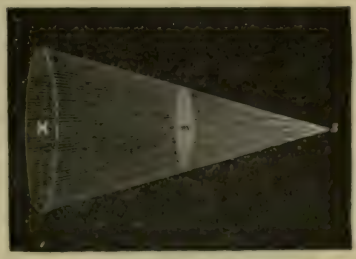

FIG. 153.—Intensity of light at different distances from the source.

Suppose L to be the rate per second at which luminous radiation is emitted uniformly in all directions from a point situated at the centre of a sphere of radius r cm. Since the area of the sphere is $4\pi r^2$, the quantity of radiation falling on each square centimetre of the sphere will be $L \div 4\pi r^2$. This quantity is termed the **intensity of illumination** of the surface; and it is evident that the illumination varies inversely as the square of the distance.

FIG. 154.—Shadow thrown by a luminous point.

Shadows.—Let P (Fig. 154) be a luminous point and S an opaque sphere intercepting the light which falls upon it. A screen held beyond S and at right angles to

the axis of the cone of rays will indicate a well-defined circular shadow of the sphere, all points of the screen within the shadow being protected totally from the rays.

If the source of light be large, as in S (Fig. 155), a more complicated shadow is obtained. Each point of S throws its own shadow cone; and by drawing the cones due to two points at the upper and lower edges of S, it is seen that only the space common to both these cones is protected completely from light; this space is indicated by the black cone terminating at the point P. When a screen AB is held between E and P, so that it is normal to the line joining the centres of S and E, there will be visible on its surface a

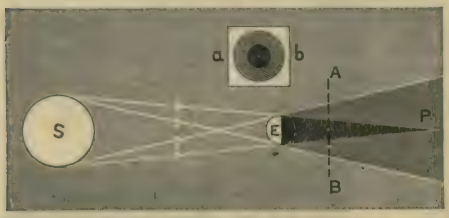

FIG. 155.—Umbra and penumbra, due to an extended source of light.

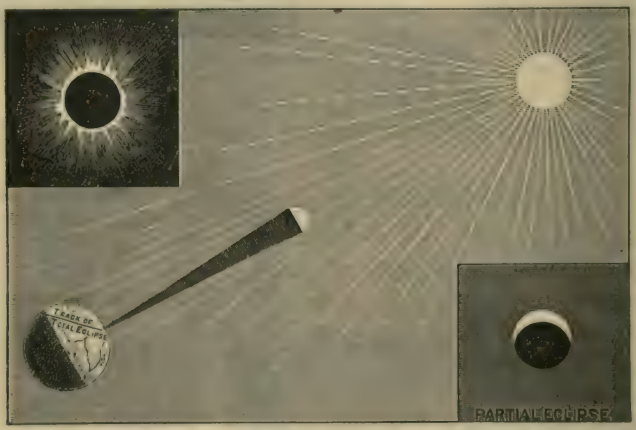

FIG. 156.—Eclipse of the sun, caused by the moon coming between the sun and the earth.

black central circle called the **umbra**, surrounded by a ring of partial shadow, termed the **penumbra**. The appearance of the shadow is shown at *ab*.

A shadow of this type is thrown by the light of the sun falling upon the moon. When this shadow falls upon the earth's surface, a *total* eclipse of the sun is seen from any point within the umbra, and a *partial* eclipse is seen from any point within the penumbra (Fig. 156). Similarly, the sun throws a shadow of the earth, and the moon is partially (or totally) eclipsed when it passes into the shadow.

Dimensions of shadow-cones.—The length of a shadow-cone, or the diameter of the cone at any point, can be determined by graphical construction or by a simple calculation. For instance,

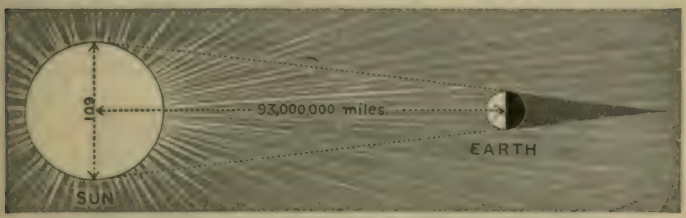

FIG. 157.—Shadow-cone of the earth.

Fig. 157 represents the shadow-cone caused by the sun shining upon the earth. The diameter of a cone at any point is directly proportional to the distance from the apex. Hence we have the relation

$$\frac{\text{Earth's diameter}}{\text{Sun's diameter}} = \frac{\text{Earth's distance from apex}}{\text{Sun's distance from apex}}.$$

If the earth's diameter be taken as unity, the sun's diameter is about 109 times greater. The earth's distance from the sun is 93,000,000 miles; and the distance (x) of the earth from the apex of the shadow may be found from the simple proportion:

$$\frac{1}{109} = \frac{x}{93{,}000{,}000 + x}.$$

The length of the earth's shadow is thus found to be 861,111 miles. Knowing the distance of the sun, the diameter can be calculated by the same principle based upon the fact that a half-penny, which is one inch in diameter, exactly covers up the sun's

disc when held at a distance of nine feet from the eye. For we have the proportion:

$$\frac{\text{Diameter of halfpenny}}{\text{Diameter of sun}} = \frac{\text{Distance of halfpenny}}{\text{Distance of sun}},$$

or, reducing inches to decimals of a mile,

$$\frac{0{\cdot}000019}{x} = \frac{0{\cdot}000019 \times 12 \times 9}{93{,}000{,}000}.$$

From this relation the sun's diameter is found to be about 860,000 miles. It is of interest to notice that the earth's shadow-cone has about the same length as the sun's diameter.

It is often easy to determine the size and shape of a shadow by a graphical construction as in the following example:

Example. A man 6 feet in height is standing 15 feet from a lamp 12 feet high: what is the length of the man's shadow upon the ground?

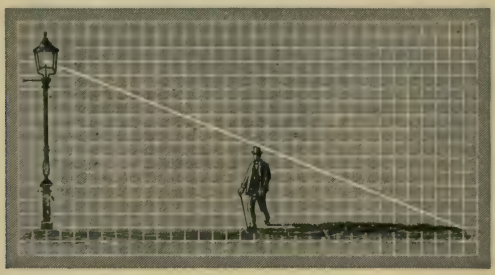

FIG. 158.—Length of shadow.

By drawing the given dimensions to scale, or on squared paper, as in Fig. 158, it will be found that the required length is 15 feet. The same result is obtained by calculation as in previous examples.

Velocity of light.—The ether-waves travel with a velocity of 186,000 miles per second. This rate of propagation has been determined in the case of light in several different ways, two of which will here be described briefly.

1. **By observations on Jupiter's satellites.**—The planet Jupiter, represented by J in Fig. 159, has several satellites or moons which revolve round Jupiter in a plane nearly coincident with that of the planet's orbit round the sun, and consequently one or other of these satellites frequently passes into the shadow of the planet thrown by the sun, and so becomes invisible to us. When the

earth E, and Jupiter J, are on the same side of the sun, let the time of disappearance of one of the moons into the planet's shadow be observed. Suppose the time at which the phenomenon should happen six months hence to be calculated by considering the period which the satellite takes to revolve around the planet. It is found that when the half-year has elapsed the

FIG. 159.—Romer's method of determining the velocity of light from observations of eclipses of Jupiter's satellites.

observed time of disappearance and the calculated time do not agree. The eclipse occurs about 1000 seconds late. This is because the earth's position relatively to Jupiter has undergone a change; it is now at E′ on the other side of its orbit, and the light-message has to travel across the orbit before reaching us. This extra journey takes 1000 seconds. The distance across the earth's orbit, that is, from E to E′, may be taken as 186,000,000 miles; and as light takes 1000 seconds to traverse this, the velocity is 186,000,000 divided by 1000 or about 186,000 miles per second.

2. **By terrestrial experiments.**—The velocity of light has been determined by various experimenters on the earth itself. We shall only describe the principle of the method employed by M. Fizeau in 1849 (Fig. 160). Two places about five miles apart were chosen and light sent from one station was reflected back to its starting point by a mirror which it struck normally at the more distant station. There was then interposed between the mirror and the source of light a toothed wheel, carefully constructed so that the width of the teeth and spaces between them were equal. This was placed near the source of light. It is clear that if the reflected light be received by a tooth of the wheel it will not reach an eye suitably placed on the same side of the wheel as the source of light. Moreover, if the wheel be rotated it can be given such a speed that there is always a tooth in the way to meet the light which has travelled to the mirror and back again. Similarly

the speed can be made of such a value that one of the spaces shall always fall in the path of light. When this is the condition of things it is easy to see that the wheel rotates through an angular distance equal to the width of a tooth in a time that the

FIG. 160.—Fizeau's apparatus for determining the velocity of light. L, source of light; *w*, *w*, toothed wheel; *m*, plain glass mirror; *m'*, silvered mirror; *f*, place where the light strikes the toothed wheel.

light travels from the wheel to the mirror and back again to the wheel. The time occupied by the wheel in rotating the angular distance can be calculated at once from the rate of rotation, while the distance from the wheel to the mirror can be measured directly.

PHOTOMETRY.

Illumination of a surface.—When light is emitted from a point at the rate L, the illumination on a surface situated at a distance d, and at right angles to the path of the rays, is equal to $L/4\pi d^2$. Suppose two luminous points, emitting light at the rates L_1 and L_2 respectively, to be situated at a distance apart; if an opaque white screen be held between the points. at right angles to the line joining them, and if the screen be at distances d_1 and d_2 from the points, the illumination on the two sides of the screen will be represented by $L_1/4\pi {d_1}^2$ and $L_2/4\pi {d_2}^2$. If the screen be moved to and fro until the two sides are equally illuminated, then

$$\frac{L_1}{4\pi {d_1}^2} = \frac{L_2}{4\pi {d_2}^2},$$

or

$$\frac{L_1}{L_2} = \frac{{d_1}^2}{{d_2}^2}.$$

This principle serves as an accurate method for comparing the luminosity of different sources of light; and this comparison is termed **Photometry**.

Candle power.—In order to give a numerical value to the illuminating power of any source of light, it is necessary to have some standard source which can be taken as a unit. It is sufficient to mention here the simplest unit available, viz. the **standard candle.** This is defined as a sperm candle, weighing six to the pound, and burning 120 grains of wax per hour. The luminosity of a candle flame is influenced by the temperature and purity of the air, and therefore this unit is not absolutely constant. The ratio between the illuminating power of any source of light and that of a standard candle is termed the **candle power** of that source.

Rumford's shadow photometer.—In this photometer (Fig. 161) an opaque rod is placed in front of a vertical screen of unglazed paper (or of ground glass) and the two sources of light are placed so as to throw separate shadows of the rod upon the screen. The shadow due to one of the sources represents a strip of the screen which is illuminated by light from the other source only. The distances of the sources from the screen are varied until the shadows are equally dark, and then the above equation can be applied for comparing the luminosity. It is not necessary to have a completely dark room for experiments with this photometer.

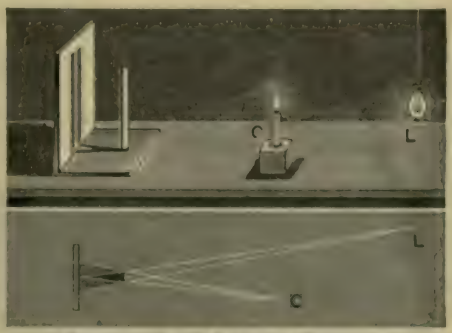

FIG. 161.—Rumford's shadow photometer.

EXPT. 168.—**Law of inverse squares.** Pin a piece of white paper upon a drawing board to act as a screen. Fix the drawing board at right angles to a table in a darkened room. In front of the screen place a vertical rod about 1 to 2 cm. in diameter (a retort stand will do). Beyond this, place to one side a candle fixed on a block of

wood, and to the other side four candles, fixed close together on a second block of wood. Notice that two shadows of the upright rod appear on the screen. Move the candles near each other so that the two shadows of the rod touch but do not overlap. Notice that one shadow, that cast by the four candles, is darker than the other. The latter shadow is illuminated by one candle, the other is illuminated by four candles. Now, move back the four candles until the shadows appear equally dark, that is, in equal contrast with the bright part of the screen. Then the four candles give just as much light to the screen as the single candle. Measure the distance of the single candle, and the mean distance of the four.

It will be found that the latter distance is twice the former, thus showing that at double the distance of a single candle four candles are required to give the same illumination.

EXPT. 169.—**Candle-power.** Using a standard candle, determine the candle-power of (i) an ordinary wax candle, (ii) a paraffin lamp, and (iii) a fish-tail gas burner. For this comparison, use the relation $L_1/L_2 = d_1^2/d_2^2$.

Bunsen's grease-spot photometer.—In this photometer the two sources of light are compared by placing them on either side of a screen having a grease spot in it. Its use depends on the fact that a grease spot equally illuminated on either side has the same brightness as the general surface. The light lost by transmission

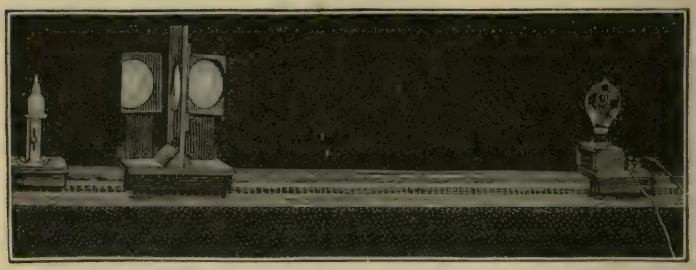

FIG. 162.—Bunsen's grease-spot photometer.

through the translucent or semi-transparent spot in one direction is compensated for by the equal transmission in the opposite direction. The eye is indifferent whether the light it receives is due to a reflection from white paper or to transmission through a translucent spot. It observes that the brightness of the grease

spot is equal to that of the rest of the surface. The intensities of the two lights are again proportional to the squares of their distances from the screen.

EXPT. 170.—**Unequal illumination.** Obtain a piece of white paper. Make a grease spot in the centre. Allow a light to shine on the paper. Observe that the grease spot is darker than the surrounding surface. Observe the paper by transmitted light. Notice that the grease spot is now brighter than the general surface.

EXPT. 171.—**Equal illumination.** Use the paper as a screen and illuminate one side of it by means of a candle and the other with a lamp. Move the candle and lamp until the grease spot is barely distinguishable in point of brightness from the white surface near it. Measure the distance of the candle and lamp from the grease spot. Using the law of inverse squares, calculate the luminosity of the lamp in terms of the candle.

EXERCISES ON CHAPTER XVIII.

1. Describe a pin-hole camera, and explain, illustrating your answer by a diagram, how the image of a luminous object is formed by it.

What experiment would you perform to show why it is that the image first becomes blurred and then disappears when the size of the hole is increased gradually?

2. Three candles are placed quite close together in a row at the centre of a room, and a wooden rod is held in a vertical position at a distance of about a foot from the candles. Explain, giving diagrams, why it is that as the rod is moved in a circle round the candles the shadow cast on the walls is in some positions sharp and in others very ill defined.

3. The sun shines through a crack in the shutter of a darkened room. A person inside the room says that he sees a ray of light entering the room. Put his statement in a more accurate form. What can he really see?

4. A small opaque sphere is placed between a gas burner and a white screen, and when the gas is turned down so that the flame is very small it is found that the shadow cast on the screen is quite sharp, but on turning up the gas so that the flame is large, that the edge of the shadow is blurred. Explain the reason for this change, illustrating your answer by means of diagrams.

5. A man, $5\frac{1}{2}$ ft. high, is standing at a distance of 5 ft. from a street lamp, the flame of which is 9 ft. above a horizontal roadway. Find the length of the man's shadow.

6. Draw a diagram of the sun, moon, and earth during an eclipse of the moon. How far from the earth does its umbra extend if the diameters of the sun and earth are 800,000 miles and 7900 miles respectively, and if the distance of the earth from the sun is 92,000,000 miles?

7. The sun subtends the same angle as a halfpenny at a distance of 10 feet. Give a diagram showing the size and nature of the shadow of a halfpenny cast by the sun on a surface perpendicular to the rays at a distance of 5 feet from the halfpenny.

8. A shadow of a circular table 3 feet high and 3 feet in diameter, is cast on the ceiling of a room 10 feet high by a night light on the floor and vertically beneath one edge of the table. Give a diagram illustrating the formation of the shadow, and find its size and shape.

9. In a comparison of the luminous intensity of an incandescent electric lamp and of a standard candle, it is observed that the shadows were of equal intensity when the distances of these sources from the screen were 42 cm. and 15 cm. respectively. What was the 'candle-power' of the lamp?

10. The shadows of a vertical rod on a wall, at equal distances of 2 feet each from the rod, cast by two gas flames are observed to be equally dark when the flames are at distances of 6 and 4 feet respectively from the rod. Compare the illuminating powers of the flames.

11. Compare the intensities of the illumination produced on the floor of a room (i) when lit by a gas-lamp of 400 candle-power at a height of 16 ft., and (ii) when lit by an arc lamp of 1000 candle-power at a height of 40 ft.

12. (i) A standard candle is 210 cm. from a 16 candle-power electric lamp. Where should a screen be placed between them in order that its two sides may be illuminated equally?

(ii) Where else, along the line joining the lights, might the screen be placed and yet be illuminated equally by each source of light?

13. What is meant by the intensity of illumination on a surface? Which would give the greater intensity of illumination on the ground immediately beneath the lamp, (i) a 100 candle-power lamp at a height of 12 feet, or (ii) a similar lamp of 1200 candle-power 45 feet high?

14. How would you arrange an experiment to determine the percentage of light that is transmitted through a neutral tinted glass plate? If a plate of such glass allowed 40 per cent. of the light incident upon it to pass through, how much light would be transmitted by a plate of the same glass of four times the thickness, assuming no light to be lost by reflection at the surfaces in either case?

15. Two lamps are placed on opposite sides of a screen, and their distances from the screen so adjusted that the two faces of it are

illuminated equally. A semi-transparent sheet is then placed between one of the lamps and the screen, and it is found that the other lamp must be moved to twice its original distance from the screen in order that the two faces may be illuminated equally again. What fraction of the light falling upon it is cut off by the sheet?

16. Describe some way of comparing the candle-powers of two different sources of light.

How would you estimate by experiment the percentage of the light available which is lost by a person working in a room with a dirty window?

17. A candle is placed inside a box in a dark room. A small hole is cut in one side of the box, and a sheet of paper is held a short distance in front of the hole. Describe and explain the appearance seen on the paper.

18. A candle flame is placed on one side of an opaque screen in which there is a small hole. Explain—and illustrate your explanation by a diagram—why it is that if a piece of white paper be held on the other side of the screen an image of the candle is seen on it.

Is the result affected by (i) the shape, (ii) the size of the hole?

19. Describe a method by means of which the velocity of light has been determined.

CHAPTER XIX.

REFLECTION AT PLANE SURFACES.

Reflection and refraction.—When rays of light fall upon the surface of any medium, some may be reflected back from the surface, and others may be transmitted through the medium. If transmitted, they may be more or less absorbed, as in the case of a translucent medium, or as in the case of coloured water or glass, which rapidly absorb light of certain colours but readily transmit those of other colours. As a rule, the transmitted rays are **bent**, or **refracted**, away from their original path.

Reflection of light rays may happen in two ways, either **regularly** or **irregularly**. In the first case, they are turned back according to simple rules, while in the second there is no uniformity about the direction of reflection. In Fig. 163 the surface AB of a medium which is denser than air is supposed to be a perfect plane, and it gives rise to regular reflection; the surface BC is uneven, and the reflection from it is irregular. The diagram shows also how some of the rays may be transmitted through the denser medium.

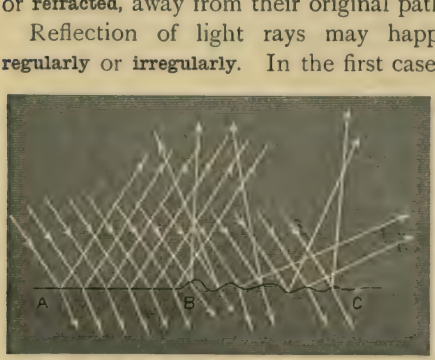

FIG. 163.—Regular and irregular reflection.

The page on which this explanation is printed appears to be white because—owing to the roughness of the paper—of the

irregular reflection of the light which falls upon it. Or, if we powder a sheet of glass, the powder seems to be white for a similar reason; there are, in these and similar cases, many surfaces formed from which irregular reflection takes place.

Laws of reflection of light.—Light is reflected regularly from a plane mirror—that is, a flat reflecting surface. Such a mirror can be made from a variety of substances, but the most common is bright metal or silvered glass.

The angle between the path of the incident ray and the normal to the mirror (that is a perpendicular to the mirror at the point where the ray strikes it) at the point of incidence is termed the **angle of incidence.** The angle between the normal and the path of the reflected ray is termed the **angle of reflection.**

There is a definite connection between the angles of incidence and reflection, and it can be expressed as follows:

The paths of the incident and reflected rays are (i) **in the same plane as the normal at the point of incidence,** (ii) **at equal angles on opposite sides of the normal.**

It can be proved by experiment also that when a wave strikes a reflecting surface *normally*, *i.e.* having travelled along the normal, it is reflected back upon the same line.

EXPT. 172.—**Pin method of proving laws of reflection.** Fasten a sheet of white paper on a drawing board, and support in a vertical position on the paper a strip (about 2 inches by 1 inch) of good plane mirror AB (Fig. 164). (The mirror may be supported by fixing a small wooden cube to the back of the mirror with wax.) Fix two pins, P and Q, vertically in the board and approximately in the positions shown. View the images of these pins by looking in the direction SR, and move the eye until the image of Q overlaps that of P. Keep the eye in this position, and insert two other pins at points such as R and S, and so that these pins together with the images of P and Q all appear to be in one straight line.

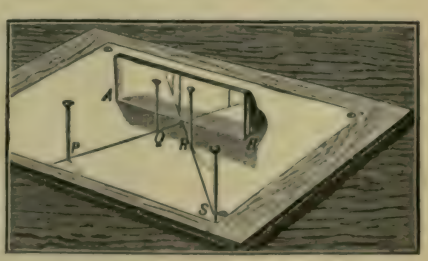

FIG. 164.—Pin method of determining the laws of reflection of light.

Draw a pencil line at the back of the mirror. Remove the mirror; draw the lines PQ and SR and produce them until they meet (Fig. 165). They should meet at a point O approximately on the line of the silvered surface. Draw a line ON normal to the mirror at O. Measure the angle of incidence PON, and the angle of reflection SON, and compare them. Repeat the experiment two or three times, with different angle of incidence in each case.

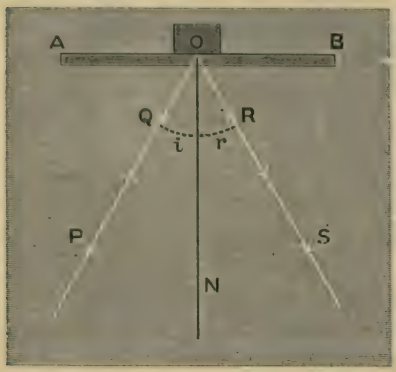

FIG. 165.—Construction to illustrate laws of reflection of light.

Equality of distances of object and of image from a plane reflecting surface.—The two rules of reflection enable the formation of an image by a plane mirror to be understood easily. By a simple geometrical construction it can be shown that the image of a point is at the same distance behind a plane mirror that the point itself is in front of the reflecting surface.

Let MM′ (Fig. 166) be a horizontal section through a plane mirror, and O a bright point like the head of a pin. Let O*r* and O*p* be any two rays from O which are reflected by the mirror along the paths *pq* and *rs*. When an eye is situated at e_1 these rays give rise to the impression that the source of the rays is at some point I behind the mirror. Draw *pn*, a normal to the mirror at the point *p*.

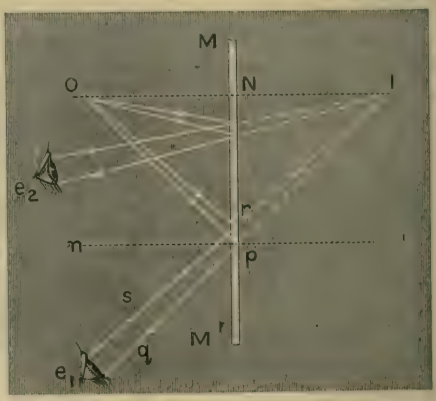

FIG. 166.—Reflection by a plane mirror.

Since $\angle Opn = \angle qpn$,

then $\angle Opr = \angle qpM' = \angle Ipr$.

Similarly $\angle OrM = \angle IrM$;

therefore $\angle Orp = \angle Irp$.

Hence, the triangles Orp and Irp are equal; and $Or = Ir$. Join the points O and I by a line passing through the mirror at N. Then, in the triangles OrN and IrN, the sides Or and Ir are equal, the base Nr is common, and $\angle OrN = \angle IrN$. Hence, the triangles are equal in all respects (*Euc.* I. iv.). Therefore,

$$ON = IN; \text{ and } \angle ONr = \angle INr.$$

Hence, **the image is situated on the normal to the mirror drawn from the source of the rays; and its distance behind the mirror is equal to that of the object in front of the mirror.**

Fig. 166 represents also how the image will appear to be situated at the same point I when the eye is in any other position, such as e_2.

The above result applies equally to the formation of the images of objects, which can be regarded as accumulations of small material particles to which the construction given already for a point may be applied.

FIG. 167.—Expt. 173.

EXPT. 173.—**Image and object.** Support a strip of plane mirror AB (Fig. 167) in a vertical position on a sheet of paper, and fix a pin vertically at O. View the image of the pin in the direction PQ, and insert pins at P and Q. Similarly, view the image in the direction RS, and insert pins at R and S. The image is situated somewhere along the line PQ produced, and also somewhere along the line SR produced. Hence it can only be at the point where these lines intersect. Remove the mirror, and produce the lines PQ and SR to meet at I. Draw normals to the mirror from I and O. These normals should be in the same straight line, and equal in length.

EXPT. 174.—**Second method of finding an image of an object.** Use the same mirror as before. If it be 1 inch high, use pins about 2 inches long. Insert a pin at O (Fig. 167), and view its image just to one side

of the normal. Fix a pin O′ in the paper behind the mirror so that the upper part of this pin may appear to be a continuation of the image of the pin at O. Move the eye slightly to the right and then to the left; if the upper part of O′ appears to move relatively to the image of O in the *same* direction as the eye, then O′ is *too far* from the mirror. Adjust the position of O′ until, from whatever direction it is viewed, it always appears to be continuous with the image of O. *The pin* O′ *then occupies the position of the image of* O.

This second method of determining the position of an image of an object is known as the **Parallax method.** Parallax may be defined as the apparent change in the position of an object due to a change in the position of the observer. It may be demonstrated thus: Place two rods, A and B, vertically, in line with the eye, and with B just behind A. If the eye is moved to the *right*, then B appears to move to the *right* of A (or, it may be said that A appears to move to the *left* of B). If the eye is moved to the *left*, then B appears to move to the left. **The further object appears to move in the same direction as the observer's eye.** If B is vertically over A, then they appear to be in the same straight line, whatever the direction of view may be.

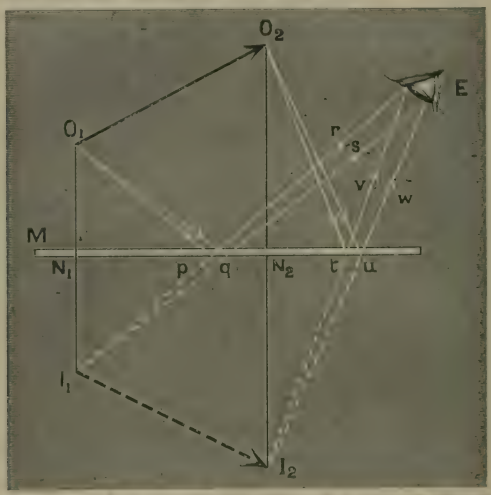

FIG. 168.—Image of an extended object, viewed by means of a plane mirror.

Image of an object in front of a plane mirror.—Let M (Fig. 168) represent a plane mirror, and O_1O_2 an object placed in front of the mirror. From O_1 and O_2 draw normals, O_1N_1

and O_2N_2, to the reflecting surface; and produce these to points I_1 and I_2, such that $N_1I_1 = N_1O_1$, and $N_2I_2 = N_2O_2$. Then I_1I_2 is the image of the object O_1O_2. The path of the pencil of rays which originates from O_1 and enters an eye situated at E is obtained by drawing the pencil (I_1pr and I_1qs) which *appears* to originate from the point I_1, thus locating the points p and q on the reflecting surface at which the rays from O_1 are reflected. Join O_1p and O_1q. The rays O_1pr and O_1qs represent the pencil of rays by which the image of O_1 is seen. In a similar manner, the rays O_2tv and O_2uw represent how the rays from O_2 are reflected so as to give the image at I_2.

FIG. 169.—Lateral inversion, due to a plane mirror.

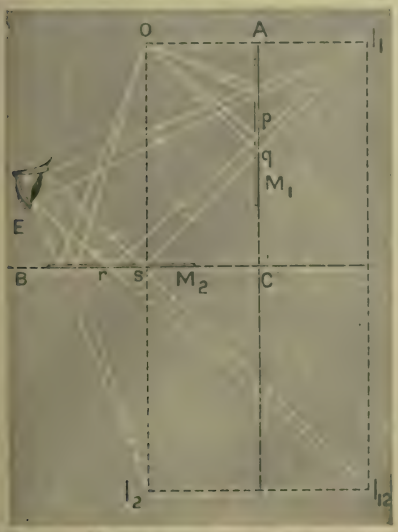

FIG. 170.—Multiple images, due to two mirrors at 90°.

It is evident from this diagram, that an object, when viewed by means of a plane mirror, undergoes **lateral inversion**: the left-hand side of the object when viewed directly, appears to be the right-hand side when viewed by means of the mirror. This result is rendered more apparent when a page of printed matter is viewed in a mirror; each letter, and the sequence of the letters, is reversed sideways (Fig. 169).

Images formed by inclined mirrors.—Fig. 170 represents two plane mirrors, M_1 and M_2, placed vertically with their reflecting

surfaces coinciding respectively with two lines, AC and BC, which are at right angles to each other. When a luminous point O, situated within the angle ACB, is viewed by an eye at E, three images are visible. The images I_1 and I_2 are obtained each by *one* reflection, from the mirrors M_1 and M_2 respectively. The third image I_{12} is obtained by rays which are reflected *twice* before reaching the eye. This third image may be regarded either as an image of I_1 obtained by means of the mirror M_2,

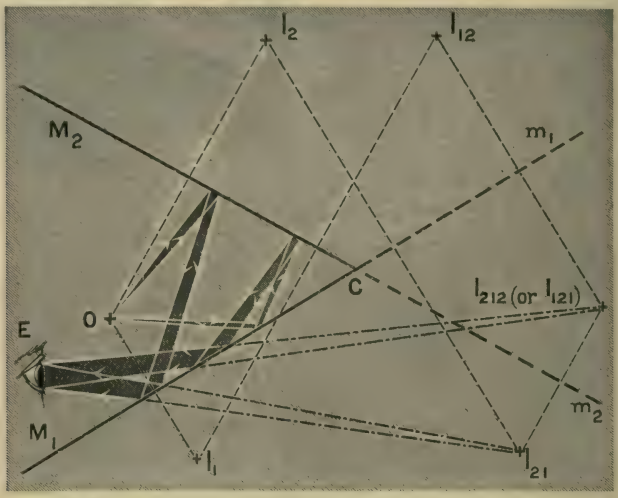

FIG. 171.—Multiple images in two mirrors inclined at 60°.

or as an image of I_2 obtained by means of the mirror M_1. The paths of the rays which give rise to this image, as seen by an eye at E, are indicated by the lines O*pr* and O*qs*.

When the angle between the mirrors is diminished, the number of images is increased. It can be proved that when the angle is $\theta°$, the number of images is $\frac{360°}{\theta°} - 1$; thus, when the angle is 60°, the number of images is 5.

Fig. 171 indicates the position of the five images, of a luminous point O, obtained by means of two mirrors CM_1 and CM_2 inclined at 60°. It is important to remember that any image gives rise to a second image when it is situated in *front* of a reflecting surface or in front of the surface produced. For this

reason the reflecting surfaces are produced in the directions Cm_1 and Cm_2. The images I_1 and I_2 are given by single reflections from the mirrors CM_1 and CM_2 respectively. The image I_1 gives rise to a second image I_{12} by reflection from the mirror CM_2, and the image I_2 gives rise to a second image I_{21} by reflection from the mirror CM_1; in both these cases, the rays of light undergo *two* reflections before reaching the eye E, and the diagram indicates the path of the rays which give rise to the image I_{21}. Finally, the images I_{21} and I_{12} give coincident images at I_{212}, the former by means of the mirror CM_2, and the latter by means of the mirror CM_1; the path of the rays which give rise to this image are shown in the diagram, and it will be noticed that the rays undergo *three* reflections before reaching the eye. No further images are generated, since I_{212} is situated *behind* the reflecting surfaces of both mirrors. It is interesting to bear in mind that the object and all the images are situated on the circumference of a circle described round C as a centre.

EXPT. 175.—**Positions of images formed by inclined mirrors.** Fasten a sheet of paper on a drawing board, and draw two lines inclined at 60°. Place two strips of mirror, similar to those used in Expt. 172, so that their reflecting surfaces coincide with these lines. Fix a pin vertically at any point such as O (Fig. 171). In the first instance, count the number of images. Determine the position of each image by the method of parallax. With centre C, and radius CO, describe a circle; observe whether each image is situated on the circumference. Make a careful diagram showing the paths of the rays giving rise to at least two of the images, assuming the eye to be placed in any given position.

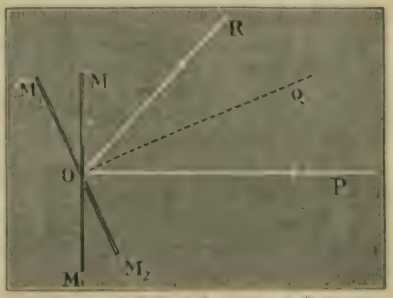

FIG. 172.—Effect of rotating a mirror.

Rotation of a mirror.—When a mirror rotates, the angle through which the reflected ray moves is twice the angle through which the mirror moves.—This fact is utilised in the **Sextant**, an instrument used by surveyors and navigators to measure the angle subtended by two distant objects.

Knowing the laws of reflection, the effect of rotating a mirror can be arrived at very easily by simple geometry. Thus, let MM

(Fig. 172) be a plane mirror, capable of rotation round an axis at O, and let PO be a ray of light incident on the mirror at O and normal to its surface. The ray is reflected back along its previous path.

If the mirror be rotated round O into the position M_1M_2, the angle through which it is rotated is equal to POQ, since this is the angle between the normals to the mirror in its initial and final positions.

When the mirror is at M_1M_2, the reflected ray is represented by OR ; and, by the rotation of the mirror, the reflected ray has been rotated through the angle POR.

By the fundamental law of reflection,

$$\angle QOR = \angle POQ,$$

or

$$\angle POR = 2\angle POQ.$$

Hence, the angle through which the reflected ray moves is twice the angle through which the mirror moves.

EXERCISES ON CHAPTER XIX.

1. State the two laws in accordance with which a ray of light is reflected by a smooth surface, and describe experiments by which you would demonstrate the truth of each of these laws.

2. What is an inverted image? If the capital letter F were drawn on paper and held in front of a mirror, how would you have to draw the letter on the paper and how hold the paper, in order that the image of the letter in the mirror should present its ordinary aspect?

3. By moving a fragment of looking-glass, a boy finds that he can throw images of the sun up and down the walls and ceiling of a room. Where must he stand to be able to do this? Show by a diagram that the angle through which the image moves is twice as large as the angle through which the boy moves the glass.

4. What deviation is produced by reflection at a plane surface when the angle of incidence is 60°?

5. Make a measured drawing showing the positions of all the images of a luminous object placed between two plane mirrors inclined at 45°.

6. Two plane mirrors are inclined at an angle of 60° ; give a carefully drawn diagram showing the position of the images of a luminous object placed so that the plane through it and the line of intersection of the mirrors makes an angle of 45° with one of the mirrors.

7. A mirror hangs on one of the walls of a room ; show by means of carefully drawn diagrams that an observer will see by reflection more and more of the room behind him as he approaches the mirror.

8. State the laws of reflection.

Prove that a man can see the whole of his person in a mirror the length of which is half his own height.

9. Draw a diagram to show what images of an object, placed between two plane mirrors that make an angle of 50° with one another, are formed by reflection.

10. How would you show by experiment that the rays from a luminous point proceed after reflection by a plane mirror as if they come from a point as far behind the mirror as the luminous point is in front?

11. A horizontal beam of parallel light is reflected by a vertical plane mirror. The mirror, remaining vertical, is turned through a small angle. What is the relation between the angle through which the mirror is turned and the angle between the initial and final directions of the reflected beam? Give reasons.

12. Two mirrors are placed at right angles to one another. In a plane at right angles to the mirrors an arrow is placed, lying on the bisector of the angle between the mirrors. Draw a diagram to show the positions of the images of the arrow formed by the mirrors.

CHAPTER XX.

REFRACTION AT PLANE SURFACES.

Refraction of light.—Up to the present, rays of light have been supposed to be moving through a uniform medium. When this is so, as has been seen, light travels in straight lines, and, if it meets a reflecting surface, it is turned back, according to the laws described in the last chapter. When, however, the light passes from one medium into another of a different optical density, the propagation of the wave is usually no longer rectilinear: except in those cases where the incident ray is normal to the surface of separation between the two media, the passage from one medium into the other is accompanied by a bending of its path. This bending is known as **refraction**, and the ray is said to be **refracted**.

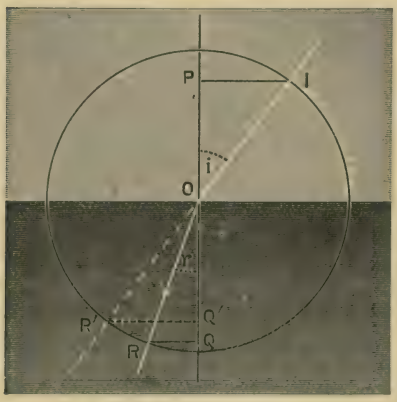

FIG. 173.—Law of refraction.

Laws of refraction.—In Fig. 173 the shaded lower part of the diagram represents a denser medium than the unshaded upper portion. The word denser is used here, and in similar connections, to mean optically denser, and must not be confused with what has been said of the density of bodies in Chapter IV. Let

IO represent a ray passing from the rarer to the denser medium, or the ray incident on the surface of the denser medium at O. The angle IO makes with the normal at O is the **angle of incidence**. The ray is bent; instead of continuing its course in a straight line along OR', it is refracted and travels in the direction of OR, which represents the refracted ray, the angle ROQ being the **angle of refraction**. The angle ROR', which represents the amount the ray has been turned out of its original direction, is termed the **angle of deviation**. Let a circle be described with the centre O and any convenient radius, and from the points where it cuts the incident and refracted rays, let perpendiculars be drawn to the normal as in Fig. 173. A perpendicular is also dropped from the point R'. It is clear from geometry that the perpendicular R'Q' is equal to the perpendicular IP. The ratio between the lengths of R'Q' and RQ is constant for the same two media, *e.g.* air and water, whatever the angle of incidence. This ratio is called the **index of refraction** of the denser medium, or the **refractive index**, and it is usually denoted by the symbol μ. Its value for air and water is about $\frac{4}{3}$; for air and glass approximately $\frac{3}{2}$, depending upon the kind of glass.

The laws of refraction may be expressed thus:

1. **The incident and refracted rays are on opposite sides of the normal at the point of incidence, and in the same plane as the normal.**

2. **If a circle be described about the point of incidence, and perpendiculars be dropped upon the normal from the intersections of this circle with the incident and refracted rays, the ratio of the lengths of these perpendiculars is constant for any two given media.**

The student who is familiar with the elements of trigonometry will find the following statement of the second law to be more convenient: **The ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant for any two given media.** This may be written $\mu = \sin i / \sin r$, where i and r are the angles of incidence and of refraction respectively, and μ is the refractive index. It can be proved experimentally that if the direction in which the light is travelling be reversed, the path of the ray remains unaltered. Consequently, Fig. 173 may be used to represent the refraction of a ray proceeding from a dense into a rarer medium; and it is evident that the refracted ray is bent away from the normal. If, in this case,

i'' and r' represent the angles of incidence and refraction respectively, then

$$\frac{\sin i''}{\sin r'} = \frac{\sin r}{\sin i} = \frac{1}{\mu}, \quad \text{.....(1)}$$

or $$\mu = \sin r'/\sin i''.$$

EXPT. 176.—**The refractive index of glass.** Spread a sheet of white paper upon a drawing board, and draw a straight line on the paper. Lay a thick glass slab (about $10 \times 8 \times 1{\cdot}5$ cm.; *rectangular edges*) on the paper with its edge AB (Fig. 174) upon this line. Close to the edges CD and AB insert pins P and Q, so that the line PQ is oblique to AB. Look at P *through the block*, and move the eye until Q covers the image of P. Insert a third pin, R, in such a position that it appears to be in line with P and Q. Remove the block; draw the normal MM'; with Q as centre, describe a circle, 4-5 cm. radius. Drop perpendiculars LM and KM', and measure their lengths. The ratio LM/KM' is the *refractive index* (μ) of glass.

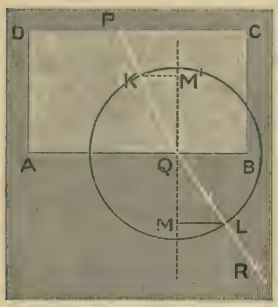

FIG. 174.—Refraction by glass.

Replace the block, alter the position of the pin P, and make another determination of μ.

EXPT. 177.—**The refractive index of water.** In Fig. 175 E is the vertical edge of a strip of paper fastened outside a glass trough ABCD, containing water. Look at this edge, through the trough of water, and trace the path of the rays, outside the trough, by means of two pins, Q and R. Determine the refractive index by the same construction as used in previous experiment.

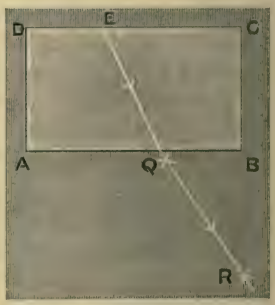

FIG. 175.—Refraction by water.

In speaking of the *refractive index* of any material it is always assumed that the ray of light proceeds *from* air *into* the material, and not in the reverse direction.

It will be shown subsequently that when light is proceeding from a dense into a rarer medium there is no emergent ray if the

angle of incidence exceeds a certain limit; in such a case all the light is reflected back into the denser medium.

Geometrical construction for refraction at a plane surface.—Let IO (Fig. 176) be an incident ray at O, passing into an optically denser medium of which the refractive index is $\frac{4}{3}$. With O as centre describe a circle, of any radius, cutting the incident ray at I. From I draw IN perpendicular to the refracting surface. Divide ON into *four* equal parts, and measure off ON′ equal to *three* of these parts. Draw N′R perpendicular to the refracting surface, and cutting the circle at R. OR is the direction of the refracted ray.

FIG. 176.—Refraction of incident ray passing into an optically denser medium.

The correctness of this construction is evident, since, by definition, the refractive index is equal to the ratio of the perpendiculars IM and RM′; and

$$\frac{IM}{RM'} = \frac{ON}{ON'} = \frac{4}{3}.$$

FIG. 177.—Refraction of incident ray passing into a less dense medium.

Fig. 177 represents the construction when the ray passes from the denser into the less dense medium. In this case ON is divided into three equal parts, and ON′ is measured off equal to four of these parts.

When the angle i is increased slightly, so that N′ coincides with the circumference of the circle, the refracted ray will coincide with the surface of separation. Any further increase in the angle of incidence will result in all the light being reflected back into the denser medium. This

limiting value for the angle of incidence is termed the **critical angle** for the denser medium.

The critical angle.—When a ray of light is proceeding from a dense into a rarer medium, the maximum value which the angle of refraction can have is 90°. Since $\sin 90° = 1$, the greatest possible value of the angle of incidence such that a refracted ray may be formed is given by the equation

$$\frac{\sin i''}{\sin 90°} = \frac{1}{\mu},$$

or

$$\sin i'' = \frac{1}{\mu}.$$

Hence, for any dense medium **the critical angle is the angle of which the sine is equal to the reciprocal of the refractive index of the medium.** The critical angle for water is about 49°, and for crown glass 42°.

EXPT. 178.—**The critical angle for glass.** Fasten a sheet of paper on a drawing-board, and place a glass prism ABC (Fig. 178) on the paper. Trace the outline of the prism on the paper. Fix a pin O vertically in the board and touching the face AC of the prism. Place the eye to the right of B and look along the edge BA; move the eye gradually towards C and look for an image of the pin O reflected from the surface AB. When the line of sight corresponds with a direction p_1p_2 the image will become dim; and if the eye be moved slightly towards C the image will disappear. Mark with pins, p_1 and p_2, the direction in which the image is barely visible. Remove the prism; join the points p_1 and p_2, and produce the lines so as to cut BC at Q. Since AB acts as a mirror, the rays after reflection will proceed as though coming from a point i which is behind AB and at a distance equal to that of O in front of AB. Join iQ. The intersection of this line with AB gives the point P from which the rays are reflected. Join OP. Thus

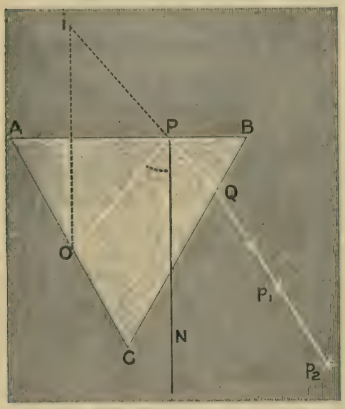

FIG. 178.—Determination of the critical angle for glass.

OPQ, the path of the rays within the glass, is obtained. Draw a normal PN at P. Then OPN is the critical angle.

In the preceding experiment, the rays of light by means of which the pin is seen when viewed from the direction P_1P_2 are said to undergo **total internal reflection.** This phenomenon suggests how a glass prism, the angles of which are 45°, 45°, and 90°, may be used as a mirror (Fig. 179). The prism is placed so that the incident rays are normal to one of the mutually perpendicular faces. The angle of incidence on the hypotenuse is therefore 45°; and, as this is greater than the critical angle, the rays are totally reflected and emerge from the prism normally to the third face. This device serves as a more perfect mirror than a glass plate which is silvered at the back, since, in the latter case, some of the light is reflected from the front surface, and some from the silvered surface, resulting in the image being made more or less indistinct.

FIG. 179.—Total internal reflection by a prism.

Fig. 180 represents rays of light proceeding from a luminous point A beneath the surface of water. Only those rays the

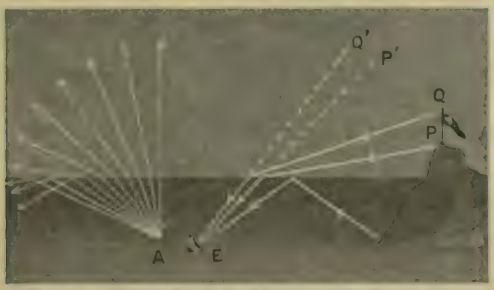

FIG. 180.—Effects due to total internal reflection in water.

angle of incidence of which is less than the critical angle emerge above the surface of the water; all others are totally reflected into the water. Similarly, to an eye situated at E below the

surface, objects above the water appear to be concentrated within a cone, of which the angle is equal to twice the critical angle; thus, the point P will appear to be situated at P′, and the point Q at Q′. Outside the boundaries of this cone objects beneath the surface of the water will be visible.

Refraction through a transparent plate.—The term plate is used to describe a slab of material with parallel faces. In Fig. 181, SP represents a ray incident at P on the side of a plate ABCD. Within the plate the ray is deviated along the path PT. On emerging from the plate the ray is deviated along the path QR; the deviation in this case being equal in amount, but opposite in direction, to the first deviation at P. Hence **in a transparent plate there is no final angular divergence, but the path of the ray is laterally displaced.** It is evident that the angle e of emergence must be equal to the angle i of incidence, since the angles r and r' are equal.

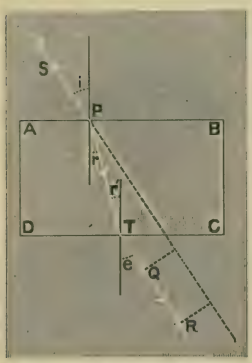

FIG. 181.—Refraction through a transparent slab.

EXPT. 179.—**Parallelism of emergent and incident rays.** Place a slab of glass ABCD (Fig. 181) upon a sheet of paper and trace its outline with a pencil. Insert a pin P near the edge AB. View the pin in an oblique direction, such as QR, and insert pins Q and R in line with the image of P. Finally, insert a pin S in line with R, Q, and P. Remove the slab; draw a line through S and P, also through R and Q cutting the slab at T. Join PT. Draw perpendiculars from Q and R on to SP produced. Measure these perpendiculars and note whether they have the same length.

The image of a point by refraction at a plane surface.—So far we have considered the refraction of a single ray only. In order to find the relative positions of an object and of its image formed by refraction at a plane surface we must consider the refraction of a small *pencil*, or assemblage, of rays.

Suppose O (Fig. 182) to be a luminous point situated in the denser of two media (X and Y), and let OCD be a narrow pencil of rays with its axis ON normal to the surface separating the two

media. The paths of the rays after deviation appear to an eye looking downwards along the normal to diverge from a point I. The paths of the rays can be determined by geometry, if the refractive index between the two media is known.

If μ be the refractive index when the rays pass from X into Y, then $1/\mu$ is the refractive index when the rays pass from Y into X. By drawing a normal at C, it is evident that the angle CON is the angle of incidence of the ray OC, and that CIN is the angle of refraction. Then

$$\frac{1}{\mu} = \frac{\sin \text{CON}}{\sin \text{CIN}} = \frac{\text{CN}}{\text{CO}} \Big/ \frac{\text{CN}}{\text{CI}} = \frac{\text{CI}}{\text{CO}},$$

or

$$\mu = \frac{\text{CO}}{\text{CI}}.$$

But when the pencil is *very narrow*, then CO and CI are approximately equal to NO and NI respectively. Hence

$$\mu = \frac{\text{NO}}{\text{NI}}.$$

Or, **the ratio between the true distance and the apparent distance of the point below the surface of separation is equal to the refractive index.**

Hence, as the refractive index of water is 4/3, an observer looking straight down upon the surface of water sees an object in the water at three-quarters its actual distance from the surface.

This result is only true for small pencils of rays which are nearly *normal* to the surface. If the pencil entering the eye strikes the separating surface obliquely, the apparent position of the point is altered considerably; thus, in Fig. 182, the rays O*c* and O*d* are refracted so as to appear to diverge from a point I_1.

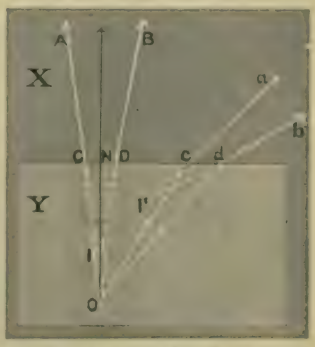

FIG. 182.—Apparent change of position caused by refraction.

EXPT. 180.—**Refractive index of water, by measurement of apparent depth.** Fill a deep glass cylinder with water, and drop to the bottom of the cylinder a small opaque object (*e.g.* a short piece of thick copper wire). Clamp a narrow

glass tube, drawn out to a jet, vertically above the surface of the water. Connect the tube to the gas supply, fix it so that the jet is horizontal, and light the gas at the jet adjusting the supply so that a *small* yellow flame is obtained. View the arrangement vertically downwards and observe whether there is any parallax between the immersed object and the image of the flame reflected from the

FIG. 183.—Apparent bending and shortening of an immersed rod.

surface of the water. Adjust the distance of the jet above the water until there is no parallax. In this position the apparent depth of immersion of the object must coincide with the position of the flame's image, and the distance of the latter below the surface is equal necessarily to the vertical height of the jet above the surface. Take the necessary measurements, and calculate the refractive index.

This apparent displacement of a source of light immersed in a denser medium explains why a straight rod partly immersed slantwise in water (Fig. 183) appears to be bent and shortened just at the surface of the water, when the stick is viewed from one side.

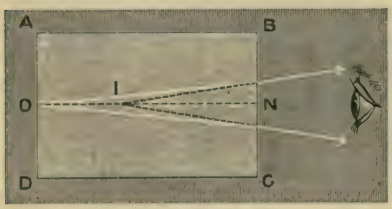

FIG. 184.—Expt. 181.

EXPT. 181.—**Refractive index of glass, by locating the image due to refraction at a plane surface.** Lay a slab of glass ABCD (Fig. 184) on a sheet of paper, and fix a pin vertically at O in contact with one edge of the

glass. View the pin through the glass from a position along the normal ON and beyond the opposite edge. That portion of the pin which is seen through the glass will appear to be at some point I. Determine the position of this point by holding another pin vertically with its point touching the glass and moving the point to-and-fro along ON until a position is found where the movable pin appears to remain continuous with the image of O when the eye is moved slightly to right and left.* Measure the distance IN and ON, and calculate the refractive index of glass.

Refraction through a prism.—When a wedge-shaped piece of glass, or a prism, as it is called in optics, is interposed in the path of a ray of light from a small hole in the cap of a lantern, it is easy to see, by watching the image of the hole on a screen, that the image moves in a direction towards the base of the prism. This is because the ray is bent by its passage through the prism, so that on its emergence from the glass it continues in a new path inclined towards the base of the prism. The amount of bending experienced by the ray of light depends, among other conditions, upon the angle between the inclined sides of the prism meeting in its edge, or **the angle of the prism**, as it is called.

In Fig. 185 let the triangle ABC represent a section of the prism at right angles to its faces, such as we should see by looking at the

Fig. 185.—Refraction of a ray of light through a prism.

end of it. Suppose DE is a ray of light striking the face AB of the prism. The light on entering the prism passes from the air

* For description of this parallax method, see p. 252.

into the glass, or *from a rarer into a denser medium*, and is bent *towards* a line drawn perpendicular to the face of the prism at the point where the ray of light strikes it. It consequently travels along the line EE′ until it reaches the face AC of the prism. Here it passes from the glass into the air, *i.e. from a denser into a rarer medium*, and is, in such circumstances, bent *from* the perpendicular, and travels along the line E′D′. In every such passage through a prism it is noticed that the light is always bent or refracted towards the thick part of the prism.

EXPT. 182.—**Pin method of tracing deviation by a prism.** Stand a prism upright, that is, upon one of its ends, upon a piece of white paper. Stick two pins into the paper in positions such as D and E (Fig. 185), E being as close as possible to the face of the prism. Place two more, E′, D′, on the opposite side of the prism, so that the four appear in a straight line when looking through the prism. Draw the outline of the prism ABC, and then take away the prism and the pins and connect the pin-holes as shown in the diagram. It will be found that the ray is bent towards the base of the prism both when it enters and emerges. Measure the angle FGD′, which is termed the *angle of deviation* (*d*); also measure the angle of the prism at A.

Repeat the experiment, using a different angle of incidence. Note whether the deviation is the same as before; and note also that the path of the ray within the prism is not necessarily parallel to the base BC of the prism.

Minimum deviation.—When experiments similar to Expt. 182 are carried out with prisms of different angles and of different kinds of glass, it can be proved that the deviation of a ray passing through a prism depends upon

(i) the refracting angle of the prism,
(ii) the material of the prism,
(iii) the angle of incidence of the ray entering the prism, and
(iv) the nature of the incident light.

With a given prism there is one angle of incidence for which the angle of deviation is a minimum; and it can be proved, both theoretically and by experiment, that the deviation has a minimum value when the angle of emergence is equal to the angle of incidence (or, in other words, if the two sides of the prism are equal in length, when the path of the ray within the prism is parallel to the base of the prism).

EXPT. 183.—**Minimum deviation and calculation of the refractive index of a prism.** Place a prism ABC (Fig. 185) upon a sheet of paper, and fix two pins in positions corresponding to D and E. View the two pins through the prism in the direction D'E'. Slowly rotate the prism round the point A, and in either direction: notice that the line D'E' of the image varies, and that there is *one position* of the prism in which D'E' approaches most nearly to the direction GF of the incident ray. Mark the emergent ray by means of pins D' and E', and trace the outline of the prism on the paper. Remove the prism and pins.

Draw the incident ray DE and the emergent ray E'D'. Join EE'. The path of the ray is represented by DEE'D'. Note whether EE' is parallel to the base BC. Produce DE to any point F, and D'E' to G. Measure the angle of deviation (d). Draw normals at the points E and E', and measure the angles of incidence (i) and of emergence (e). The angle A of the prism is equal, necessarily, to the angle (a) between the normals at E and E'.

Notice that, from Fig. 185, the angle a is equal to twice the angle r of refraction; hence

$$r = \frac{a}{2}.$$

Also, since the angle e of emergence is equal to the angle i of incidence,

$$\angle GEE' = \angle GE'E$$

and $$d = 2(i - r) = 2i - a;$$

or $$i = \tfrac{1}{2}(d + a).$$

It has been explained previously that the refractive index (μ) is equal to the ratio $\sin i / \sin r$; hence

$$\mu = \frac{\sin i}{\sin r} = \sin \frac{d + a}{2} \Big/ \sin \frac{a}{2}.$$

From the measurements obtained in the experiment calculate, by means of this formula, the refractive index of the glass prism.

Graphical diagrams.—The effect of refraction upon the course of a ray can be determined easily by the construction of graphical diagrams based upon the principles described already. It is only necessary to bear in mind that when a ray passes from one medium to another of different optical density, perpendiculars should be drawn to the normal at equal distances on the rays from the point of incidence and their lengths should be in the ratio of the refractive index. In the denser medium the normal

must represent always the smaller number of the ratio given as the refractive index. The following examples illustrate the construction of two diagrams of this kind:

EXAMPLE.—1. A ray of light is incident at an angle of 50° to the surface of a glass plate which is 5 cm. thick. The refractive index of the glass is 1·5. Determine graphically the path of the emergent ray, and measure the lateral displacement which the ray undergoes.

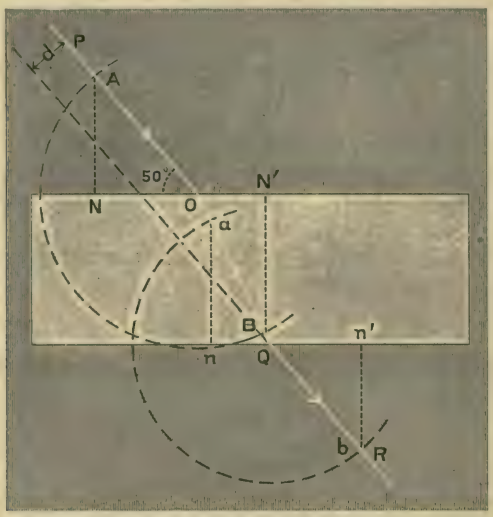

FIG. 186.—Geometrical construction for the path of a ray through a transparent slab.

Let PO (Fig. 186) be the incident ray, and O the point of incidence. With centre O, and with any convenient radius, describe a circle which cuts the incident ray at A.

From A draw a normal AN to the surface of the glass. Measure ON, and mark off a distance ON′ such that $ON = (1{\cdot}5 \times ON')$. From N′ draw the normal N′B cutting the circle previously described at B. Join OB, and produce it if necessary to meet the lower surface of the plate at Q. OQ is the *refracted ray.*

With centre Q, and with any convenient radius, describe a circle which cuts the ray OQ at *a*. From *a* draw the normal *an*. Measure Q*n*, and mark off a distance Q*n*′ such that $Qn' = (1{\cdot}5 \times Qn)$. From *n*′ draw the normal *n*′*b* cutting the circle at *b*. Join Q*b*. Then Q*b* is the direction of the *emergent ray.*

The lateral displacement is determined by producing the ray QR backwards and measuring the perpendicular distance *d* between the rays QR and PO.

EXAMPLE.—2. An equilateral hollow glass prism (with very thin walls) is filled with carbon bisulphide. Trace the path of a ray of light which falls on the prism making an angle of incidence of 60° on the side of the normal away from the apex of the prism. Measure the deviation. [Refractive index of carbon bisulphide = 1·7.]

In Fig. 187 let PO be the incident ray. Measure off, along the face AB, distances OD and OE such that OD = 1·7 × OE. From D and E draw normals to the face of the prism.

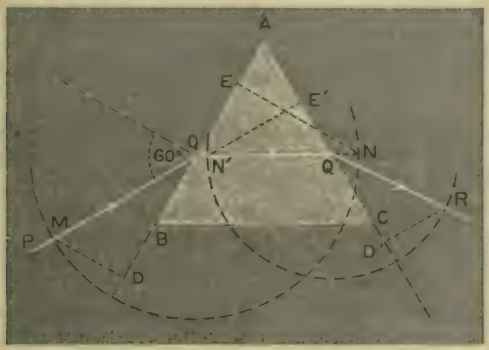

FIG. 187.—Geometrical construction for the path of a ray through a prism.

Let the normal from D cut the incident ray at the point M. With centre O, and radius OM, describe a circle. If this circle cuts the normal, drawn from E, at the point N, then the line ON is the path of the ray through the prism.

Let Q be the point of emergence. Measure off, along the face AC, distances QD′ and QE′, such that QD′ = 1·7 × QE′. From E′ draw a normal, intersecting the path OQ at N′. With centre Q, and radius QN′, describe a circle, cutting the normal from D′ at the point R. The line QR is the path of the emergent ray.

The angle of deviation is found to be 57° approximately.

EXERCISES ON CHAPTER XX.

1. A bright bead is placed at the bottom of a basin of water, and a person stands in such a position that he can just see it over the edge of the basin. While he is looking, the water is drawn off. How will this affect his view?

Draw a diagram showing the direction of a ray of light passing from the bead through the water and the air in each case.

2. A thick layer of transparent liquid floats on the surface of water. Trace the course of a ray of light from an object immersed in the water through the floating liquid to the air.

3. Describe an experiment to show the path of a ray of light which passes obliquely through a thick plate of glass. Illustrate your answer by a sketch in which you indicate clearly the path of the ray in the air before it enters the glass, in the glass, and in the air beyond the glass.

4. An upright post is fixed in the bottom of a pond which is three feet deep; the top of the post is three feet above the water. How will the post appear to an eye about the level of the top of the post and four or five feet away from it?

Draw a figure to illustrate your answer.

What will be seen as the eye moves further and further back from the post?

5. A fish swims in a glass tank; a person whose eye is above the level of the water seems to see two fish. Draw a diagram to illustrate this, and give any explanations you think necessary.

6. A vertical straight wire is viewed so that part is seen directly and part through a thick plate of glass held vertically. Describe the apparent changes in the position of the portion seen through the glass when this is slowly turned round a vertical axis.

7. A glass cube is placed over a pencil mark on a sheet of paper. The mark is viewed through the cube. Explain, by means of a diagram, the apparent position of the pencil mark.

8. What is meant by the refractive index of a substance? Being provided with a thick piece of glass from a box of weights, some pins, a sheet of white paper, a pencil and graduated ruler, how would you determine the index of refraction of the glass?

9. State the laws of the refraction of light. Explain why a tank of water viewed from above appears shallower than it really is.

10. Explain, by the aid of a diagram, why a pole appears to be bent when it is thrust into water in a slanting position.

11. A ray of light is incident at an angle of 30° on one face of a glass plate, 3 inches thick, the index of refraction being 1·5. Give a diagram to scale showing the refracted and emergent rays. Show that the emergent ray is parallel to the incident ray, and measure the perpendicular distance between them.

12. Explain the terms *critical angle* and *total reflection.* A glass cube of two inch edge stands upon a horizontal table; represent in plan, in a diagram drawn approximately to scale, the path of a horizontal ray of light which falls upon a vertical face of the cube and is afterwards totally reflected at a surface of contact of glass

and air. Assume the refractive index of the glass with respect to air to be $\frac{3}{2}$.

13. Two faces of a glass block are parallel and 3 inches apart; and a ray of light strikes one of them at an angle of incidence of 60°. The refractive index of the glass is 1·5. Show in a diagram the directions of the incident ray, refracted ray, and emergent ray. Measure the amount of displacement caused by the glass and record it.

14. The critical angle for a certain medium is 45°. What is its refractive index?

15. Trace the path of a ray of light which falls upon one of the perpendicular faces of an isosceles right-angled glass prism ($\mu = 3/2$) at an angle of 70° with that surface. Trace the ray until it emerges from that surface.

16. A layer of water, 2 inches deep, lies upon a slab of glass which is 1·5 inches thick and of which the lower surface is silvered. Trace the path of a ray which strikes the surface of the water at an angle of 45° ($_{\text{air}}\mu_{\text{water}} = 4/3$; $_{\text{water}}\mu_{\text{glass}} = 9/8$).

17. Find, by geometry, the critical angles for ice, glass and carbon bisulphide, having given that the refractive indices of these substances are 1·3, 1·5, and 1·7 respectively.

18. Show, by geometry, why a ray of light striking one of the perpendicular faces of a right-angled isosceles glass prism along a normal cannot emerge from the hypotenuse face. Trace its path. ($\mu = 3/2$.)

19. ABCD is the square section of a glass cube ($\mu = 3/2$). P is a luminous point in AC produced, and 1 inch distant from C. PQ is a ray of light striking the side BC at R, so that $\angle APR$ is 30°. Trace the path of the ray until it emerges from the glass. [Make AB = $3\frac{1}{2}$ inches.]

20. An equilateral hollow glass prism is filled with carbon bisulphide ($\mu = 1·7$). Trace the path of a ray which is incident upon the prism at 30° with the surface.

21. A ray of light is incident at an angle of 30° on one face of an equilateral prism. If the path of the light through the prism is parallel to the base, find the direction of the emergent ray, and the total deviation of the ray after passing through the prism.

22. A dot is made on a piece of paper, and a prism is laid on the paper over the dot. An eye in certain positions now seems to see two dots. Draw a diagram to explain this.

23. A hollow glass prism, and full of air, is immersed in a glass tank full of water. Make a diagram showing rays of light passing through both the water and the prism.

24. The minimum deviation produced by a hollow prism filled with a certain liquid is 30°. If the refracting angle of the prism is 60°, what is the index of refraction of the liquid?

25. Explain clearly how you would determine by experiment the index of refraction of a glass plate. Why is the apparent depth of a pool less than its actual depth?

26. State the laws of the refraction of light.

A ray of light, falling on one face of a plate of glass 10 cm. in thickness, makes an angle of 60° with the normal. Determine the point on the other face at which the ray will emerge from the glass. [The index of refraction of the glass is 1·5.]

27. State the laws of refraction of light.

A ray of light is incident on the surface of water at an angle of 45°. Find by a diagram the direction of the refracted ray. [The index of refraction of water is 1·33.]

28. A ray of light in a prism, which has an angle of 60°, makes an angle of 30° with the face of the prism. Find the direction of the emergent ray. [The index of refraction of the glass is 1·4.]

29. State the laws of refraction of light.

A ray of light is incident at 60° to the normal upon a polished glass surface. The refracted ray makes an angle of 90° with the reflected ray. Find, graphically or by calculation, the refractive index of the glass.

30. A plane mirror lies horizontally. The image of a point lying above the mirror is viewed by an eye looking at an angle of 45° with the mirror, the height of the eye above the mirror being twice the height of the point. Draw a diagram to indicate the path of two rays proceeding from the point to the eye.

Explain whether the eye would still see the image of the point in the same direction if the mirror lay at the bottom of a tank, and if water were poured into the tank (*a*) so as to just cover the point, and (*b*) so as to reach up to the eye.

31. Draw a diagram of a prism with an angle of 45°, and trace completely the course of a ray of light, which, lying in a plane at right angles to the axis of the prism, falls on the prism making an angle of incidence of 30° on the side of the normal away from the apex of the prism.

From your diagram measure the deviation. [Refractive index of the glass = 1·5.]

32. An equilateral hollow glass prism is filled with glycerine. Determine graphically the minimum deviation of a ray passing through the prism. [Refractive index of glycerine = 1·47.]

33. The refractive index of crown glass is 1·53, and its critical angle is 41° approximately. Determine geometrically the minimum angle of incidence of a ray of light which falls upon one face of an equilateral glass prism and undergoes total internal reflection from the other face.

CHAPTER XXI.

REFLECTION AT SPHERICAL SURFACES.

Spherical mirrors.—A spherical mirror is a mirror having the form of part of the surface of a sphere. It may be either **concave** or **convex**, the former if the reflection takes place from the hollow side, the latter if from the bulging side.

Spherical mirrors may be considered as made up of an infinite number of very small plane facets having negligible curvature, and each such element may be regarded as a small plane mirror. It is known, from geometry, that every radius of a circle is at right angles to the tangent at the point where it cuts the circle. Hence, the normals to all these plane facets will pass, like the radii of a circle, through one point which is the centre of the sphere from which the mirror has been derived. This point is called the **centre of curvature** of the mirror.

Figs. 188 and 189 indicate how a concave mirror and a convex mirror respectively, each consisting of a number of small plane facets, will affect the path of the rays of a diverging pencil of rays originating from a luminous point O. For the sake of clearness each facet is represented with only one ray reflected from its middle point. The path of each ray after reflection is obtained by drawing the normal to the reflecting surface, and by making the angle of reflection r equal to the angle of incidence i. In the case of the concave mirror (Fig. 188) all the rays, after reflection, pass through a point I, which is called the **image** of the luminous point O ; and the position of this image could be determined by holding a piece of white paper, so as to intercept the reflected rays, and altering its position until the brightly illuminated portion of its surface is restricted to a point. In the case of the convex

mirror (Fig. 189), the rays after reflection *appear* to proceed from a point I on the further side of the mirror. To an eye situated on the left-hand side of the mirror the image of the luminous point O will appear to be situated at I.

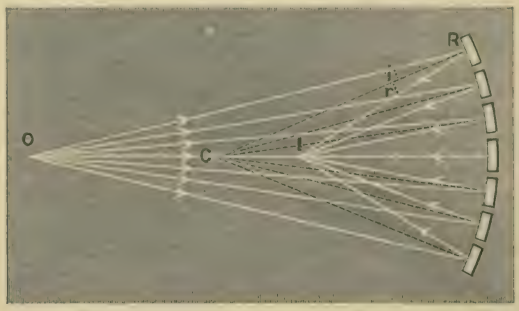

FIG. 188.—The principle of a concave mirror.

It is evident that **when a luminous point is situated at the centre of curvature of a concave mirror all the rays of light from it are reflected back along the lines of incidence, and the image is formed in the same position as the luminous point.**

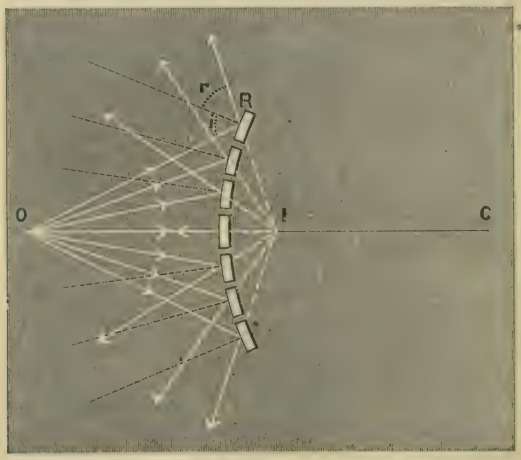

FIG. 189.—The principle of a convex mirror.

In any spherical mirror, the distance from the centre of curvature to the reflecting surface is the **radius of curvature**. Thus,

in Fig. 190, C is the centre of curvature and CM, CR, CM′, are all radii of curvature. MM′ is called the diameter or aperture of the mirror; and P, the centre of the reflecting surface, is often called the **pole** or **apex** of the mirror. A line going through the pole and centre of curvature is the **principal or optic axis** of the mirror, any other radius produced being a **secondary axis.**

Principal focus, and focal length.—(i) Let MPM′ (Fig. 190) be a concave mirror, of which C is the centre of curvature. Let IR be a single ray of a very narrow pencil of rays parallel to the principal axis of the mirror; and let R be the point of incidence on the reflecting surface. Join CR. The line CR is normal to the surface of the mirror at R. Make the angle CR*i* equal to the angle IRC. Then, by the law of reflection, R*i* is the path of the reflected ray. Let R*i* cut the principal axis at F. Then,

since $\angle CRI = \angle CRF$,

and $\angle CRI = \angle RCF$,

$\angle CRF = \angle RCF$.

Hence, $FR = FC$.

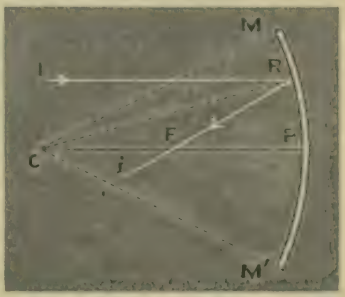

FIG. 190.—Principal focus of a concave mirror.

But, if the pencil of rays be very narrow, the point R coincides approximately with the point P; and, therefore, FR = FP approximately.

Hence, $FP = FC$.

The same reasoning will apply to all other rays in the pencil; and, therefore, **all the rays after reflection will pass through a point F which is midway between the centre of curvature and the pole of the mirror.**

(ii) Fig. 191 represents the corresponding case for a convex mirror, where IR is a single ray of a pencil parallel to the principal axis. If C*n* is the normal to the reflecting surface at R, then $\angle IRn = \angle nRi$. But, by geometry, $\angle IRn = \angle FCR$, and $\angle NRi = \angle FRC$.

Hence, $\angle FCR = \angle FRC$,

and $FC = FR$.

But, since the pencil of rays is supposed to be very narrow, FR = FP.

$\therefore FC = FP$.

Hence, **all the rays after reflection from a convex mirror appear to pass through a point F which is midway between the centre of curvature and the pole of the mirror.**

In each case the point F is termed the **principal focus** of the mirror, and the distance of this point from the mirror is termed

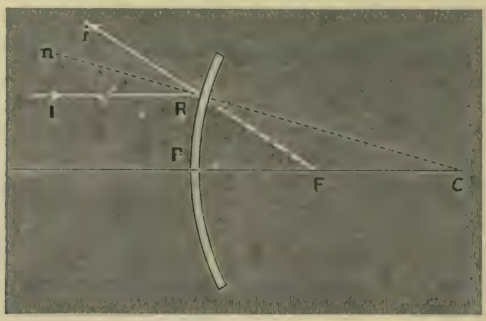

FIG. 191.—Principal focus of a convex mirror.

the **focal length** of the mirror. The following definitions are of fundamental importance :

Principal focus.—**When a narrow beam of rays parallel to the principal axis of a concave mirror is reflected from the surface of the mirror, the rays converge to, or diverge from, a point on the principal axis. This point is termed the principal focus.**

Focal length.—**The distance of the principal focus from the pole of the mirror is termed the focal length of the mirror.**

It is evident, from the preceding paragraphs, that **the focal length of a spherical mirror is equal numerically to one half the radius of curvature.**

Real and virtual foci.—In the case of a convex mirror (Fig. 191) the reflected rays only *appear* to pass through the principal focus, and, in order to distinguish this case from that in which the rays actually pass through the principal focus, it is usual to term the former a **virtual** focus, and the latter a **real** focus.

The same terms are used in all such cases, whether the incident pencil of rays is parallel, or converging, or diverging.

Position and nature of the image of an object, due to reflection from a spherical mirror.—In determining by a graphical method the position of the image of an object, use is made of the fact that the paths of certain rays after reflection from the mirror are known,

providing that the positions of the centre of curvature and of the principal focus are given. Thus

(i) **A ray which passes through the centre of curvature of a mirror returns by the same path after reflection.**

(ii) **Rays parallel to the principal axis of a mirror pass through the principal focus after reflection.**

And, since the path traversed remains unaltered when the direction of the ray is reversed,

(iii) **Rays which pass through the principal focus assume a direction parallel to the principal axis after reflection.**

In practice, it is sufficient to make use of two only of these general principles, since the point of intersection of the paths of two rays after reflection suffice to determine the position of the image.

The diagrams obtained by such methods afford information as to (i) the *position* of the image, (ii) its *nature*, whether real or virtual, and (iii) its *size*.

In calculating the position of the image, by means of the **general equation for mirrors**, which will be deduced in a subsequent paragraph, it is necessary to apply always the following rule as to the signs *plus* and *minus*: Distances measured from the mirror *towards* the source of light are **positive**; but when measured from the mirror and *away from* the source of light they are to be considered **negative**. As a result of this rule, the focal length of a concave mirror is always positive, and that of a convex mirror is always negative (Figs. 190 and 191).

Real images due to a concave mirror.—In Fig. 192 let OB be an object situated further from a concave mirror than its centre

FIG. 192.—Real image, by means of a concave mirror.

of curvature C. A ray BR parallel to the principal axis will be reflected so as to pass through the principal focus F; and a ray

BC, passing through the centre of curvature, will be reflected back along its own path. The point M, where these reflected rays intersect, is the position of the image of the point B. It may be proved, in a similar manner, that the image of any other point on OB is situated along the line IM. Hence IM is the image of the object OB.

Since the reflected rays actually pass through the image, the image is **real**; and it is evident from Fig. 192 that the image is **inverted**.

Conjugate foci.—The nature and position of an image formed by a mirror depends upon the position of the object. Consider a point in any given position upon the principal axis. Rays diverge from it to the mirror and converge to form an image which may be real or virtual, but in either case the object and image may be regarded as interchangeable. Points at which an object and image may thus change places are known as **conjugate foci**. When the image is real, as for instance, when the object is beyond the centre of curvature of a concave mirror, the places actually can be changed, so that the image is formed where the object was previously.

Equation for mirrors.—In determining, by calculation, the relationship between the positions of the object and image, and the focal length of the mirror, it is necessary to remember that the mirror is supposed to be very narrow, and that the pole P of the mirror coincides with the point D, which is the base of the normal drawn from R to the principal axis.

If u = the distance, PO, of the object from the mirror,
v = „ PI, „ image „
r = the radius of curvature, CP,
and f = the focal length, FP, of the mirror;
then $OB/IM = CO/CI$, and $RD/IM = FD/FI$.
But, since $OB = RD$, $OB/IM = RD/IM$;
hence $CO/CI = FD/FI = FP/FI$.
But $CO = (u - 2f)$; $CI = (2f - v)$; $FP = f$; and $FI = (v - f)$;
hence $$\frac{u - 2f}{2f - v} = \frac{f}{v - f},$$
or $$uv - uf - 2fv + 2f^2 = 2f^2 - fv,$$
or $$uv = uf + vf.$$

From which the **general equation for mirrors** is derived, namely,

$$\frac{1}{\text{focal length}} = \frac{1}{\text{distance of image}} + \frac{1}{\text{distance of object}},$$

or

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}.$$

Virtual images, due to concave mirrors. In Fig. 193 let OB be an object situated nearer to a concave mirror than its principal focus. Consider the two rays BR and BR′ proceeding from the point B. Since the path of the former passes through the

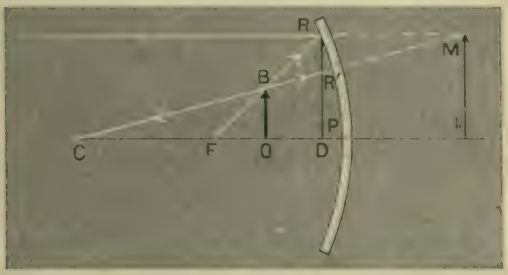

FIG. 193.—Virtual image, by means of a concave mirror.

principal focus F, its direction after reflection will be parallel to the principal axis; and, since the path of the ray BR′ passes through the centre of curvature C, it will be reflected back along its previous path. The point M where these reflected rays intersect is behind the mirror. Hence IM is the image of the object. Since the reflected rays only *appear* to originate from M, the image is **virtual**; it is evident, also, from the diagram that the image is **upright** and **enlarged**.

It is evident, from geometry, that

$$OB/IM = OC/IC, \text{ and } OB/RD = OF/DF.$$

But

$$IM = RD;$$

$$\therefore\ OC/IC = OF/DF = OF/PF \text{ (approximately).}$$

If the symbols u, v, and f have the same meaning as on p. 280, then $OC = (2f - u)$; and, since PI is measured in the negative direction, $IC = 2f + (-v) = (2f - v)$; also, $OF = (f - u)$ and $PF = f$.

Hence, $$\frac{2f-u}{2f-v}=\frac{f-u}{f},$$

or $$uv=uf+vf,$$

or $$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}.$$

This equation is identical with the general equation deduced in the previous paragraph.

Virtual images, due to convex mirrors. In Fig. 194 the object OB is situated in front of a convex mirror; and, as in the previous

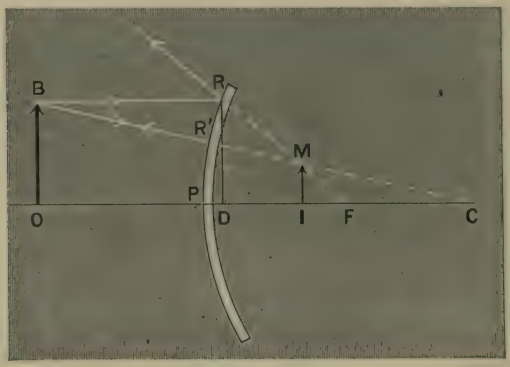

FIG. 194.—Virtual image, by means of a convex mirror.

cases, the position of the image IM is determined by the point of intersection of the paths, after reflection, of the two rays BR and BR'. It is evident that the image is **virtual, erect,** and **diminished.**

Since $OB/IM=OC/IC$, and $RD/IM=FD/FI$; $\therefore$ $OC/IC=FD/FI=FP/FI$ (approx.).

Hence, $$\frac{u+(-2f)}{-2f-(-v)}=\frac{-f}{-f-(-v)},$$

or $$\frac{u-2f}{-2f+v}=\frac{-f}{-f+v};$$

$$\therefore\ uv=fv+uf,$$

or $$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}.$$

Summary.—The following deductions are obtained from the preceding paragraphs:

(i) **Concave mirrors.**

(*a*) When the object is beyond C, the image is real, inverted, and *diminished.*

(*b*) When the object is between C and F, the image is real, inverted, and *enlarged.*

(*c*) When the object is nearer than F, the image is *virtual*, *upright*, and enlarged.

(ii) **Convex mirrors.**

In all positions of the object, the image is *virtual*, *upright*, and *diminished.*

(iii) When the proper algebraic signs are given to the numerical values of the quantities u, v, and f, the general equation $1/f = 1/v + 1/u$ may be used in all problems relating to spherical mirrors.

The conditions under which an image has any of the above characters may be summarised thus:

Real	(i)	Concave mirror: object beyond C.
	(ii)	,, ,, ,, between C and F.
Virtual	(i)	,, ,, ,, nearer than F.
	(ii)	Convex mirror: object in any position.
Erect	(i)	Concave mirror: object nearer than F.
	(ii)	Convex mirror: object in any position.
Inverted	(i)	Concave mirror; object beyond C.
	(ii)	,, ,, ,, between C and F.
Enlarged	(i)	,, ,, ,, ,, ,,
	(ii)	,, ,, ,, nearer than F.
Diminished	(i)	Concave mirror: object beyond C.
	(ii)	Convex mirror: object in any position.
Equal	(i)	Concave mirror: object at C.

EXPT. 184.—**Focal length of a concave mirror.** (i) Allow a beam of parallel rays to fall upon the mirror, and adjust the position of a piece of white cardboard so that a well-defined image is formed on its surface. The distance of the cardboard from the pole of the mirror is the focal length of the mirror. A beam of sunlight is convenient for this experiment; or a distant chimney, or window frame, may be used.

(ii) Bore a small round hole in a piece of thin cardboard, and fix two thin threads, or wires, diametrically across the hole and at right angles to each other. Fix this vertically in front of a bright flame, and allow the rays passing through the hole to fall upon the mirror. Adjust the mirror so that its principal axis makes a small angle with the axis of the incident beam. Hold a piece of white cardboard vertically and normal to the reflected beam, and adjust its position so that a well-defined image of the cross-wires is seen on its surface. Measure the distance u of the cross-wires from the mirror, and the distance v of the image from the mirror; calculate the focal length by means of the formula

$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}.$$

In this, and the following, method the surfaces of a bi-concave lens—that is, a lens having both faces concave, as in Fig. 198—may be used as mirrors, providing that the room is darkened partially.

(iii) Using the same source of light as in the previous method, adjust the position of the mirror so that a well-defined image of the cross-wires is seen upon the cardboard to which the cross-wires are attached and near to the round hole. Evidently the rays of light are reflected back along their own path, and the cross-wires must be situated at the centre of curvature of the mirror. Measure this distance; one-half of this distance is the focal length.

EXPT. 185.—**Focal length of a convex mirror.** Let O (Fig. 195) represent the position of the illuminated cross-wires, and let L be a bi-convex lens—that is, a lens having both faces convex, as in Fig. 197. The rays of light passing through L will converge to a point I, where a well-defined image will be formed upon a cardboard screen. Adjust the position of the screen until the best possible definition is obtained. Support the mirror M in a vertical position between L and I, and adjust its position until a well-defined image of the cross-wires is formed upon the cardboard at O. Since the rays are now reflected back along their own path they must fall normally upon the surface of M, and the distance IM between the screen at I and the mirror M must be the radius of curvature of the mirror. One half of this distance is the focal length.

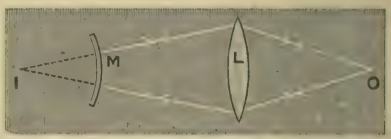

FIG. 195.—Experimental method of determining focal length of a convex mirror.

Magnification.—The ratio of the linear dimensions of the image and of the object is termed the **magnification.** Thus, in Figs. 192-194, the magnification obtained by means of the mirror is equal in each case to the ratio IM/OB.

It is convenient to express the magnification in terms of the quantities u and v. This relationship may be deduced in the following manner: In Fig. 192,

$$IM/OB = CI/CO,$$

or, expressed in words, the magnification is equal to the ratio of the distances of the image and of the object from the centre of curvature.

Since $CI = (r - v)$, and $CO = (u - r)$,

then $IM/OB = (r - v)/(u - r)$.

But $1/v + 1/u = 1/f = 2/r$,

or $1/v - 1/r = 1/r - 1/u$,

or $(r - v)/vr = (u - r)/ur$,

or $(r - v)/(u - r) = vr/ur = v/u$.

Hence, $IM/OB = v/u$,

or, **the magnification is equal to the ratio of the distances of the image and of the object from the mirror.** The same result may be deduced by means of either Fig. 193 or Fig. 194.

NUMERICAL EXAMPLE.—A luminous object 3 inches high is situated 12 inches in front of a concave mirror, the focal length of which is 9 inches. Find the position and size of the image.

Using the formula $1/v + 1/u = 1/f$; since $u = +12$ in., and $f = +9$ in.,

then $$\frac{1}{v} = \frac{1}{9} - \frac{1}{12} = \frac{1}{36},$$

or $v = +36$ in.

Hence, the image is 36 inches *in front of the mirror.*

Also, $$\frac{\text{size of image}}{\text{size of object}} = \frac{36}{12},$$

or $$\text{size of image} = 3 \times \frac{36}{12} = 9 \text{ inches.}$$

Solution of problems on mirrors by means of squared paper.—The solution, by graphical methods, of simple problems on spherical mirrors may be rendered more simple in many cases if squared paper be used for the purpose. It may suffice if suitable examples of the method are given.

EXAMPLE.—1. A luminous object, 10 cm. high, is placed in front of a concave mirror, the radius of curvature of which is 80 cm. Find the nature, size, and position of the image, when the object is (*a*) 20 cm., (*b*) 60 cm., (*c*) 110 cm. distant from the mirror.

In Fig. 196 one division of the squared paper represents 2 cm. OB is the object when situated 20 cm. from the mirror; and the position

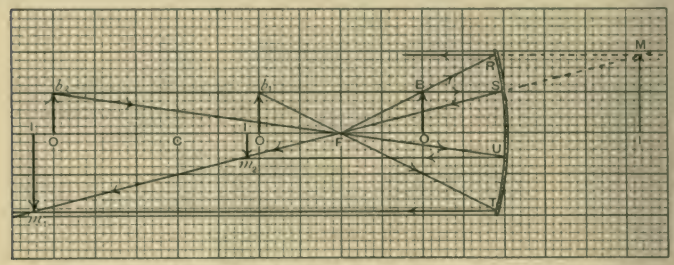

FIG. 196.—Graphical construction for image and object with a concave mirror.

of the image IM is determined by the paths of the two rays BR and BS. Similarly, the position of the image Im_1 of the object Ob_1 is determined by the rays b_1T and b_1S; and the image Im_2 of the object Ob_2 is determined by the rays b_2U and b_2S. The results obtained by measurement should be compared with those obtained by calculation from the general equation, and tabulated in the following manner:

Distance of Object.	Nature of Image.	Distance of Image.		Size of Image.	
		By Geometry.	By Calculation	By Geometry.	By Calculation
i. 20 cm.	Virtual, and upright	33·5 cm.	40 cm.	19 cm.	20 cm.
ii. 60 cm.	Real, and inverted	114·6 cm.	120 cm.	19 cm.	20 cm.
iii. 110 cm.	Real, and inverted	62·5 cm.	62·8 cm.	6 cm.	5·71 cm.

The student should endeavour to explain why the results obtained by geometry and by calculation agree more closely when the object is removed further from the mirror.

EXERCISES ON CHAPTER XXI.

1. Explain what is meant by an image. What is the difference between a real and a virtual image? In the case of a concave mirror, find the positions of the object when the image is virtual.

2. A small luminous object is placed in front of a concave spherical mirror of 12 inches focal length at distances of 3 feet, 2 feet, and 1 foot. Draw figures showing the positions and relative sizes of the images, and explain your construction.

3. A small object is placed six feet in front of a convex mirror of 3 feet radius. Give a diagram showing the nature and position of the image, and find its size relative to that of the object.

4. An object 5 cm. long is placed at a distance of 40 cm. from a convex mirror of 24 cm. focal length. Find the size and position of the image.

5. The middle of a small object is placed on the axis of a concave spherical mirror (i) half-way between the centre and the principal focus, (ii) half-way between the principal focus and the mirror. Draw diagrams to determine the position and circumstance of the image in each case.

6. Find the size of an image of the sun formed by a concave mirror 6 feet in radius, assuming that the distance of the earth from the sun is 100 times the diameter of the sun.

7. Prove that the focal length of a concave mirror is half its radius of curvature. How would you determine experimentally the focal length of such a mirror?

8. An object is placed 20 inches in front of a convex mirror of 10 inches radius. Find the position of the image, and draw a diagram to scale.

9. Explain the terms *radius of curvature*, *focal length*. How could you find by experiment the radius of curvature and the focal length of a concave mirror?

10. A mirror is fastened to the ceiling of a room and forms images of objects in the room. How can you learn from these images whether the mirror is convex or concave, and what information can you gather as to the degree of its convexity or concavity?

11. What is meant by the focal length of a mirror?
Determine graphically the position of the image formed of an object placed 30 cms. from a reflecting surface of which the radius of curvature is 40 cms.

12. Explain carefully how it is possible to find by a graphical method the position and size of the image of an object formed by reflection in a concave mirror.

A pin, 4·5 cms. long, is placed 10 cms. from a concave mirror, of which the radius of curvature is 15 cms. Find the position and size of the image.

13. Similar objects are placed close up to a plane, a concave, and a convex, reflecting surface; how would you distinguish between the surfaces from the images formed in them? How will the images change as the objects are moved away from the surfaces?

14. A concave mirror has a radius of 30 cms. Where must the object be placed so that the magnification may be 3, when the image is (*a*) real, (*b*) virtual?

15. Find the relation between the radius of curvature of a mirror and the distances of an object and its image from it. How would you test the relation by experiment?

16. Distinguish between a real and a virtual image. How would you find experimentally the position of a virtual image?

17. In an experiment with a concave mirror the following measurements of u and v were obtained:

u	v	u	v
27 cm.	77 cm.	60 cm.	30 cm.
30 „	60 „	80 „	26·7 „
40 „	40 „	100 „	25 „
50 „	33 „	120 „	24 „

Plot the readings on squared paper, and find from the curve the positions of the image when (i) $u=35$ cm., (ii) $u=70$ cm., (iii) $u=90$ cm.

CHAPTER XXII.

REFRACTION AT SPHERICAL SURFACES.

In the following paragraphs the consideration of the refraction of light rays at spherical surfaces must be limited to the deviation of rays and the formation of images by dense transparent media, which are bounded by two surfaces, of which both may be spherical, or one may be plane and the other spherical. A medium of this form is termed a **lens**.

Refraction through a lens.—Most lenses are of glass with curved surfaces, which are portions of spheres. In some lenses, however, one surface is quite plane. All lenses can be divided into two classes—**convex** or **converging**, and **concave** or **diverging**. Converging lenses are thicker in the middle than at the edges, and have the power of forming real images of objects. Concave lenses are thinner in the middle than at the edges, and are unable to form real images of objects.

Fig. 197.—A converging lens.

To understand the action of the lenses upon the course of rays of light through them, it is simplest to regard them as being built up of parts of prisms in contact, as shown in Fig. 197, where a convex lens is built up in this way. A ray of light falling upon any one of these prisms is refracted towards its thicker part, and, as in a thin lens bounded by spherical surfaces each prism has a refracting angle proportional to its distance from the axis, the rays converge toward a point, which, if the incident rays are parallel, is known as the **principal focus** of the lens, as F in Fig. 197.

Fig. 198 represents the corresponding effect due to a concave lens. The section of such a lens may be regarded as being built up of parts of prisms of gradually increasing angle, and arranged with their bases outwards. Fig. 198 represents a pencil of parallel rays converted by a concave lens into a diverging beam of rays.

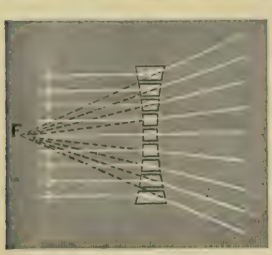

Fig. 198.—A diverging lens.

Principal axis, principal focus, and focal length.—The line connecting the centres of curvature of the two surfaces of a lens is termed the **principal axis** of the lens. The **principal focus** of a converging (or diverging) lens is the point towards which the rays converge (or from which they appear to diverge) when a pencil of rays parallel to the principal axis passes through the lens. Thus, in Figs. 197 and 198, the point F represents the principal focus. The distance from the principal focus to the centre of the lens is termed the **focal length** of the lens. With thin lenses it is sufficiently accurate to take the distance from the lens to the principal focus as the focal length.

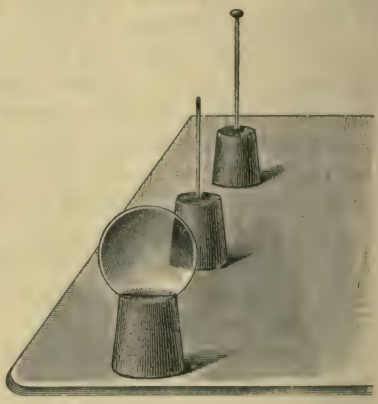

Fig. 199.—Apparatus for experiments with lenses.

Images formed by lenses.—When rays of light diverge from a luminous point on the principal axis of a lens, the rays which pass through the lens will either pass through, or appear to pass through some other point, which is termed the **image** of the luminous point. The image is **real** or **virtual** according as the rays actually pass through the point or only appear to pass through it.

The preceding paragraph does not apply in the case of a converging lens when the luminous point coincides with the

principal focus of the lens; for the emergent rays are parallel, and the image is situated at an infinitely great distance.

An image of an object seen through a lens may be either (i) *real* or *virtual*, (ii) *upright* or *inverted*, and (iii) *magnified* or *diminished*.

Much information concerning the different types of image may be derived by means of the simple appliance shown in Fig. 199. It consists of three corks which serve as supports for a lens, a needle, and a long pin: the pin should be sufficiently long for its head to be well above the top edge of the lens. If a lens of 2 inch diameter be used the pin should be $2\frac{1}{2}$ inches long.

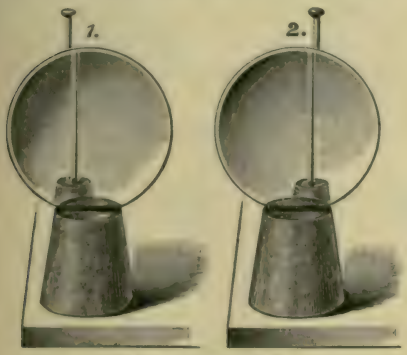

FIG. 200.—Expt. 186.

FIG. 201.—Expt. 187.

EXPT. 186.—**A concave lens.** Place the lens vertically at distances of 5, 10, 15, ... cm. in front of the pin, adjusting the height of the pin so that its head is seen well above the edge of the lens. View the image of the pin with the aid of one eye only and looking along the principal axis of the lens. Note (i) whether the image is erect or inverted; (ii) whether it is magnified or diminished; (iii) whether it is in front of, or behind, the lens; (iv) whether it is in front of, or behind, the object.

The Parallax Method (p. 252) may be applied in order to obtain information as to the position of the image, remembering that if two objects are placed in line with the eye, and the eye is then moved to the left (or right), the more distant object appears to move to the left (or right) relatively to the near object. Thus, when viewing the image of the pin through a concave lens, if the eye is moved to the left (or right), the image appears

to move towards the left (or right) edge of the lens (Fig 200); hence the image must be behind the lens. At the same time, compare the movement of the pin's head (seen above the lens) with that of the image; the former moves relatively to the latter in the same direction as that in which the eye moves; hence, the pin must be behind the image.

It is interesting to attempt to locate accurately the position of the image. The observation is not easy, but it is worth the attempt.

EXPT. 187.—**Position of image formed by a concave lens.** Place the needle vertically between the lens and the pin, and adjust its height so that its upper end is seen just above the lens. Move the eye to-and-fro and adjust the distance of the needle from the lens until *the eye of the needle* always appears to be a continuation of *the image of the pin*; the needle then occupies the position of the pin's image. Fig. 201 represents the appearance of the experiment when the eye has been moved to the left.

Enter your observations thus:

Distance of object from lens.	Is image erect or inverted?	Is it magnified or diminished?	Is it in front of, or behind, lens?	Is it in front of, or behind, object?	Distance of image from lens.
10 cm.	erect	diminished	behind	in front of	7·6 cm.

EXPT. 188.—**Convex lens.** Adjust the height of the needle so that an image of its eye can be seen through the lens. Place the lens at distances of 5, 10, 15, ... cm. in front of the needle; and, for each position, find the information required in order to fill in a Table similar to that recorded in Expt. 187.

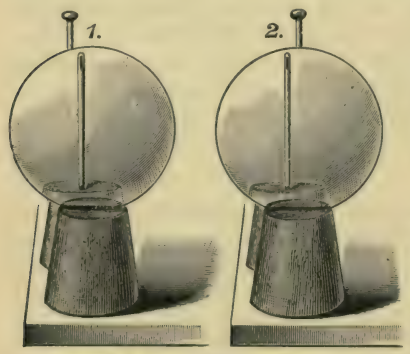

FIG. 202.—Expt. 188.

Find the position of the image of the needle by the Parallax Method, using either a long pin or a short pin according as to whether the image is behind or in front of the lens. Fig. 202, (i) and (ii), represent what would be seen when the eye is moved

to the left, and when a long pin is (i) behind, (ii) in front of the image. Notice that at one distance no distinct image can be seen, but if the distance of the object is increased slightly, an image is formed in front of the lens; in this case the image is located most readily by using a short, instead of a long, pin.

Summary.—It is evident from the preceding experiments that images due to a concave lens are always erect, diminished, and virtual (*i.e.* on the same side of the lens as the object). In the case of a convex lens, when the object is very near to the lens the image is erect, enlarged, and virtual; at one distance only, there is no image; and at greater distances the image is inverted, real, and either enlarged or diminished (depending upon the distance of the object).

The conditions under which the image assumes any of these characters may be summarised thus:—

Real	(i) Convex lens: object in any position except nearer the lens than F.
Virtual	(i) Convex lens: object nearer the lens than F. (ii) Concave lens: object in any position.
Erect	(i) Convex lens: object nearer the lens than F. (ii) Concave lens: object in any position.
Inverted	(i) Convex lens: object in any position except nearer the lens than F.
Enlarged	(i) Convex lens: object nearer the lens than F. (ii) Convex lens: object between F and 2F.
Diminished	(i) Convex lens: object at a greater distance than 2F. (ii) Concave lens: object in any position.
Equal	(i) Convex lens: object at distance 2F.

Position and nature of the images formed by lenses.—Just as in the case of images formed by reflection from a spherical mirror, it is possible to determine approximately the position of the image formed by a lens by making use of the fact that the paths traced out by certain rays are known, providing that the positions of the lens and its principal focus are known. Thus,

(i) **Rays which pass through the centre of a lens do so without change of direction.**

(ii) **Rays parallel to the principal axis of the lens are refracted so as to meet at the principal focus.**

(iii) **Conversely, rays from the principal focus of a lens assume a direction parallel to the principal axis after passing through the lens.**

In practice it is sufficient to make use of two only of these general principles, since the point of intersection of the paths after refraction of any pair of these rays suffices to determine the position of the image. The diagrams so obtained afford information as to (i) the position, (ii) the nature, and (iii) the size of the image.

In calculating the position of the image, by means of the general equation for lenses (p. 295), it is necessary to apply the following rule as to the signs *plus* and *minus*.

Rule of Signs.—Distances measured from the lens towards the source of light are positive; but when measured from the lens and away from the source of light they are to be considered negative.

By reference to Figs. 198 and 197, it is evident that the focal length of a diverging (or concave) lens is **positive**, and that of a converging (or convex) lens is **negative**.

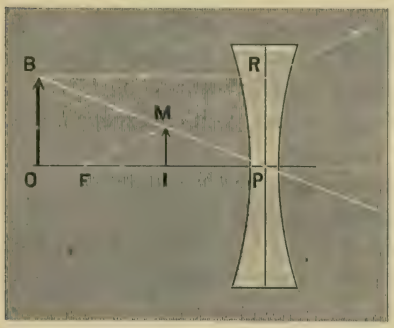

FIG. 203.—Virtual image, by means of a concave lens.

Image due to a concave lens.—In Fig. 203, let F be the principal focus of a diverging lens, of which PR is the median line. In determining by geometrical construction the paths of rays transmitted through the lens it is sufficiently approximate to assume that the lens is very thin, and that the divergence of the rays takes place abruptly at the median line.

Let OB be a luminous object. Consider the rays proceeding from the point B: one ray BR, which is parallel to the principal axis, is refracted in such a direction that it appears to originate from F. The ray BP passing through the centre of the lens undergoes approximately no deviation. The image can be

situated only at the point M, where the paths of these rays intersect. Similarly, the image of any other point of OB can be proved to be vertically under M. Hence, IM is the image of OB. It is evident that the image is **erect**, **diminished**, and **virtual**. The same result would be obtained if the object were situated nearer to the lens than the principal focus.

Relationship between the focal length and the positions of the object and image.—In Fig. 203, let $PO = u$, $PI = v$, and $PF = f$, then

$$OB/IM = PO/PI = u/v,$$

also $$PR/IM = PF/IF = f/(f - v).$$

But $$OB = PR.$$

Hence, $$u/v = f/(f - v),$$

or $$uf - uv = fv,$$

or $$uf - fv = uv,$$

which gives the **general equation for lenses**, viz.,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}, \quad \text{or} \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}.$$

This equation is applicable to all types of lenses, providing that the rule given already (p. 294), as to positive and negative values, is observed. The points at which object and image may change places without necessitating any change of construction except the reversal of the directions of the rays are known as **conjugate foci.**

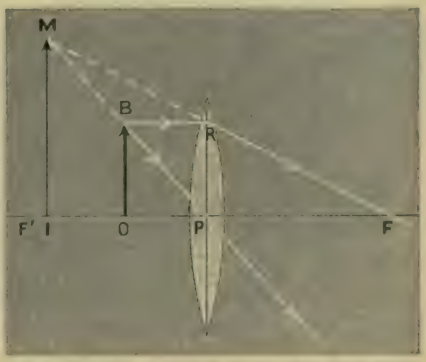

FIG. 204.—Virtual image, by means of a convex lens.

Image due to a convex lens.—The character of the image due to a convex lens depends upon the nearness of the object to the lens. In Figs. 204 and 205, let F be the principal focus of the lens. Mark off, to the left of the lens, a distance PF′ equal to the focal length PF.

CASE 1.—**Where the object is nearer to the lens than F.**

Consider the rays proceeding from a point B of a luminous object OB (Fig. 204). A ray BR, which is parallel to the principal axis, is refracted so as to pass through the principal focus F; and a ray BP, passing through the centre of the lens, is practically undeviated. The image of B must be at the point M where these paths intersect. Hence, IM represents the image of OB; and it is **erect, magnified** and **virtual.**

Fig. 204 explains the principle of the **pocket-lens** or **reading-glass** when used for slight magnification.

The general equation for lenses may be deduced from Fig. 204 in the following manner:

Since $$OB/IM = PO/PI = u/v,$$

and $$PR/IM = FP/FI = -f/(-f+v);$$

then, since $$OB = PR, \quad u/v = -f/(-f+v).$$

Hence, $$-uf + uv = -fv$$

or $$uf - fv = uv,$$

or $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

CASE 2.—**Where the object is more distant than F.**

In Fig. 205 let OB be the object. By the same construction as before, the image of the point B is situated at M; and IM is the

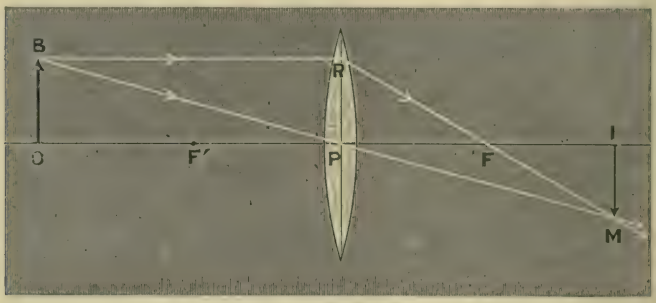

FIG. 205.—Real image, by means of a convex lens.

image of OB. Evidently the image is **real** and **inverted**; but the relative size of the image depends entirely upon the relative distances of the object and the image from the lens.

It will serve as a useful exercise for the student to deduce the general equation for lenses from Fig. 205.

Experimental determination of focal lengths.—The choice of methods of determining focal lengths of lenses depends entirely upon the apparatus available. Although a simple optical bench is desirable, yet fairly accurate results can be obtained without the aid of this appliance. The student is recommended to attempt the following methods.

Convex Lenses.

Expt. 189.—**Parallel rays.** Support the lens in a vertical position and adjust its distance from a cardboard screen so as to form on the screen a real inverted image of some distant object (*e.g.* the window bars, or a distant chimney). The rays from the distant object are practically parallel, and the image is formed at the principal focus of the lens. Measure the distance of the screen from the lens.

Expt. 190.—**Reflection method.** Bore a small circular hole through a white cardboard screen, and fix thin cross-wires across the hole with sealing-wax. Place a bright light behind and near the cross-wires. Support the lens so that the centre of the hole is on the principal axis of the lens. Place a plane mirror M (Fig. 206) close behind the lens, and adjust the distance of the lens L from the screen until an image of the cross-wires is formed on the screen P and close beside the circular hole. Evidently each ray of light is reflected back along its own path; and this can only be the case if the rays fall normally on the surface of M. Hence, the rays after passing for the first time through the lens are parallel to the principal axis, and they must therefore have originated from the principal focus. Therefore the distance f is the focal length of the lens.

Fig. 206.—Determination of the focal length of a convex lens.

Expt. 191.—**Image and object.** In this method, use is made of the general equation $1/f = 1/v - 1/u$. The student may use the simple appliances (lens and two pins or needles) as described on p. 291, determining the position of the image by the Parallax Method. Or, he may use the illuminated cross-wires (p. 284) and a second cardboard

screen. In either case, several perfectly independent observations should be made, the distance u being different in each case. In calculating the value of f it must be remembered that, if the image is real, the distance v is negative.

Concave Lenses.

Expt. 192.—**Combination method.** Select a convex lens of known focal length, and of such converging power that, when fastened to the concave lens the combination is still converging. If necessary, determine the focal length (f_1) of the convex lens, preferably by the method of Expt. 190. Fasten the two lenses together and determine the focal length (F) of the combination. The focal length (f_2) of the concave lens can then be calculated by means of the equation $1/F = 1/f_1 + 1/f_2$. When substituting the values of f_1 and F it must be remembered that the focal length of a convex lens is always negative.

Expt. 193.—**Parallax method.** Determine the focal length of a concave lens by locating the position of the virtual image of a pin or needle by the Parallax Method (Expt. 188), and applying the equation $1/f = 1/v - 1/u$.

Magnification.—The ratio of the linear dimensions of the image and the object is termed the **magnification** (m). Thus, in Figs. 203-205, the magnification due to the lens in each case is equal to the ratio IM/OB.

It is evident from either of the diagrams that

$$m = \frac{\text{IM}}{\text{OB}} = \frac{v}{u}.$$

It is more convenient in some problems to express the magnifying power of a lens in terms of u and f. This can be deduced from the general equation, thus:

Since $$\frac{1}{f} = \frac{1}{v} - \frac{1}{u},$$

then $$\frac{u}{f} = \frac{u}{v} - 1,$$

or $$\frac{u}{v} = \frac{u+f}{f}.$$

Hence, $$m = \frac{v}{u} = \frac{f}{u+f}.$$

Power of a lens.—The **power** of a lens is defined as the reciprocal of the focal length. A lens of focal length 100 cm. is recognised as having unit power; and this unit is termed the **dioptre**. It is also generally recognised that the power of a converging lens is *positive*, and that of a diverging lens is *negative*. Hence, the power of a converging lens of 50 cm. focal length is equal to $+1/0{\cdot}5 = 2$ dioptres; and that of a diverging lens of 25 cm. focal length is equal to $-1/0{\cdot}25 = -4$ dioptres.

EXAMPLES.—1. The focal length of a converging lens is 50 cm. Find the nature, position, and size of the image of an object 5 cm. long which is placed vertically (i) 25 cm., (ii) 75 cm., (iii) 125 cm. from the lens

(i) Since $f = -50$ and $u = +25$, then

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f},$$

or $$v = \frac{uf}{u+f} = \frac{25 \times (-50)}{25 - 50} = \frac{-(50 \times 25)}{-25} = +50 \text{ cm.};$$

$\therefore$ the image is virtual, and 50 cm. distant from the lens.

Also, $$m = \frac{v}{u} = \frac{+50}{+25} = 2.$$

Hence, the length of image $= 2 \times 5 = 10$ cm.

(ii) Since $u = +75$ cm., then

$$v = \frac{uf}{u+f} = \frac{75 \times (-50)}{75 - 50} = \frac{-(50 \times 75)}{25} = -150 \text{ cm.};$$

$\therefore$ the image is real, and 150 cm. distant from the lens.

Also, $$m = \frac{v}{u} = \frac{150}{75} = 2.$$

Hence, the length of the image is $(2 \times 5) = 10$ cm.

(iii) Since $u = +125$ cm., then

$$v = \frac{uf}{u+f} = \frac{125 \times (-50)}{125 - 50} = \frac{-(50 \times 125)}{75} = \frac{-250}{3} = -83{\cdot}3 \text{ cm.};$$

$\therefore$ the image is real, and 83·3 cm. distant from the lens.

Also, $$m = \frac{v}{u} = \frac{83{\cdot}3}{125} = 0{\cdot}664.$$

Hence, the length of the image is $0{\cdot}664 \times 5 = 3{\cdot}32$ cm.

2. A piece of cardboard, in which a narrow slit is cut, is placed in front of a bright light. A screen is placed vertically at a distance of 49 inches from the slit. Where must a convex lens, of focal length 6 inches, be placed so as to give on the screen a well-defined image of the slit?

The data given are $u+v=49$ inches and $f=-6$ inches.

Since the image is real and on the opposite side of the lens to that of the slit, its distance from the lens must be regarded as *negative*. Hence, in the general equation $1/v-1/u=1/f$, we may substitute $v=-(49-u)$ and $f=-6$.

$$\therefore \quad -\frac{1}{49-u}-\frac{1}{u}=-\frac{1}{6},$$

or
$$\frac{-49}{u(49-u)}=-\frac{1}{6},$$

or
$$u^2-49u+294=0,$$

or
$$(u-42)(u-7)=0.$$

Hence, $u=+42$ inches, or $+7$ inches.

A distinct image will be obtained, therefore, if the lens is placed either 42 inches or 7 inches from the slit. In the former position the image is diminished, and in the latter position enlarged.

Solution of simple problems by means of squared paper.—Squared paper may be used with advantage in solving simple problems on lenses, especially if the diagram so obtained is used as a verification of the result obtained by calculation. The following examples will indicate the method:

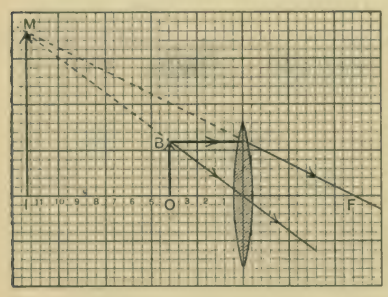

FIG. 207.—Graphical construction for image and object with convex lens.

EXAMPLES.—1. An object 3 cm. high is placed at a distance of 4 cm. from a convex lens of focal length 6 cm. Find the position and size of the image.

In Fig. 207, each division of the squared paper represents 0·5 cm. The position of the image IM is determined by the construction explained on p. 295. The diagram shows that the image is (i) virtual, (ii) approximately 11·8 cm. distant from the centre of the lens, and (iii) 11·8 cm. in height.

This result will be found to agree closely with that obtained by calculation from the general equation for lenses.

2. When an object is placed at a distance of 15 cm. from a converging lens the image formed on a screen is found to be twice the size of the object. Find the focal length of the lens.

Since the image is twice the size of the object, the distance of the former from the lens must be twice the distance of the latter from the lens. Hence, since the image is real, and therefore on the opposite side of the lens to that of the object, the distance of the screen from the object must be 45 cm.

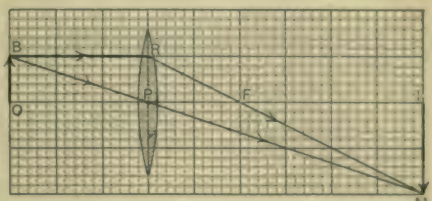

FIG. 208.—Focal length of a convex lens by graphical construction.

In Fig. 208, each division of the paper represents 1 cm.; mark off OP = 15 cm., and OI = 45 cm. Draw OB to any convenient scale, *e.g.* 5 cm., and make IM = 2OB. Draw the ray BR parallel to the principal axis, and join RM. The point F, where this ray intersects the principal axis, must be the principal focus; and PF is the focal length. By inspection PF = 9·8 cm. approximately.

EXERCISES ON CHAPTER XXII.

1. You are given a lens through which you can look, but which you are not allowed to handle. What tests would you apply in order to determine if it be concave or convex?

2. How does the image seen in a plane glass mirror differ from that seen on the ground glass of a camera, and how does each differ from the object in respect of size, and of position or inversion?

3. Define the principal foci of a thin convex lens. Draw a diagram showing the paths of the rays through such a lens from a luminous object situated on the axis of the lens at a distance of twice the focal length from its centre.

4. An object at a distance of 10 inches from a lens, when viewed through the lens appears to be at a distance of 30 inches from the lens. Find the focal length of the lens, and give a diagram showing the nature of the image.

5. A converging lens is employed to form an image of an object placed in front of it at a distance of 20 inches from the lens. If the image behind the lens is twice as large as the object, find, by a geometrical construction, the focal length of the lens.

6. A converging lens having the same focal length (5 cm.) as a concave mirror is placed with its axis coinciding with that of the

mirror, and with its centre at the centre of curvature of the mirror. Rays of light diverge from the principal focus of the lens remote from the mirror and fall on the lens. Draw a diagram to indicate their path through the lens to the mirror, and thence, after reflection, back through the lens. What difference would there be if the lens were placed with its centre at the principal focus of the mirror?

7. A pin, $\frac{1}{4}$ inch long, is placed at a distance of 3 inches from a convex lens of focal length 2 inches. Draw to scale a diagram to show the position and length of the image. Explain your construction.

8. A luminous point is situated 30 cm. in front of a lens, and an image is formed 10 cm. behind the lens. What kind of lens is used? and what is its focal length? Draw a diagram to scale, and verify the result of the calculation.

9. A pocket magnifying glass has a focal length of 5 cm. Find, by means of a diagram drawn to scale, the position and length of an object, 0·2 cm. long, placed 3 cm. from the lens.

10. The focal length of a camera lens is -20 cm. How far from the lens should the sensitized plate be in order to photograph an object 180 cm. in front of the lens? How large a surface at this distance could be photographed on a $\frac{1}{2}$-plate ($6\frac{1}{2}$ in. $\times 4\frac{3}{4}$ in.)?

11. A candle stands at a distance of 3 ft. from a wall. In what position must a convex lens of 8 in. focal length be placed between them so as to produce upon the wall a distinct image of the candle?

12. An object, 2 in. long, is placed 8 in. from a concave lens of 4-inch focal length. Find, by means of a diagram, the position and length of the image.

13. In order to find the focal length of a concave lens, it was blackened, with the exception of a circle 4 cm. in diameter at its centre. A beam of sunlight was allowed to pass through this, when it was found that an illuminated circle of 20 cm. diameter was formed on a screen held 64 cm. behind the lens and parallel to it. What was the focal length of the lens?

14. An object is placed 100 cm. from a concave lens. What must be the focal length of the lens so that a virtual image of the object can be formed at a distance of 25 cm. from the lens?

15. The focal length of a concave lens is 6 in., and a small object is placed 18 in. from the lens. Draw a diagram to scale, showing the path of the rays by which the image is formed, and determine its position.

16. A screen is fixed at a convenient distance from a lighted candle. It is found that a convex lens may be placed in two positions between the candle and screen so as to throw a distinct image of the flame on the screen. Explain this.

Being allowed to measure any distances you like, how would you determine the focal length of the lens?

17. A boy has a convex lens the focal length of which is 10 cm. How far from a screen must it be to get an image of the sun on the screen? How far from the screen must it be to get an image of a candle which is a metre from the lens? What is the power of the lens?

18. How would you find the focal length of a convex lens? An object is placed 20 cm. from a convex lens, and an inverted image is formed 4 times as large as the object. Find the focal length of the lens.

19. On a screen 1 foot behind a lens an image 6 inches long is formed of a man 5 feet 6 inches in height standing in front of the lens. Find the distance between the man and the lens, and the focal length of the latter.

20. Describe some optical (or other) method for measuring the radii of curvature of the surfaces of an ordinary lens. Upon what does the focal length of a lens depend besides the curvatures of its surfaces?

21. What procedure would you adopt to determine most correctly and easily the power of a concave lens?

22. A converging beam of light falls upon a diverging lens, the axis of the beam being coincident with the axis of the lens. Explain by means of a figure, or otherwise, the conditions for the emergent beam being convergent, parallel, or divergent.

23. A lantern slide is $3\frac{1}{4}$ inches square, and an enlarged image of it is to be formed by the aid of a lens of 6 inches focal length upon a screen 20 feet distant from the lens. What kind of lens should be used, at what distance from the slide must it be placed, and what will be the size of the image?

24. A converging lens has one of its faces plane, and the plane face is silvered. The lens is held in the path of light diverging from a small hole in a screen, the curved surface being towards the hole. When the lens is at a certain distance from the screen, a sharp image of the hole is thrown upon it. Explain, by means of a drawing, the path of the rays in their course from the hole to the image. What is the distance from the lens to the screen? Would the effect be the same if the experiment were repeated with the curved face of the lens silvered and the plane face turned towards the hole? Give reasons.

CHAPTER XXIII.

THE EYE AND OPTICAL INSTRUMENTS.

The eye.—Fig. 209 represents a horizontal cross-section through the centre of an eye. The outer coating consists of a hard thick membrane *s*, called the **sclerotic**. The front part *c* of the sclerotic is transparent, and is termed the **cornea**. A transparent lens-shaped body *o* of hard gelatinous consistency, termed the **crystalline lens**, is supported from the walls of the eye near to the cornea. The lens divides the eye into two chambers; the anterior chamber is filled with a watery liquid *a*, termed the **aqueous humour**, and the posterior is filled with a jelly-like substance *v*, termed the **vitreous humour**. Just in front of the lens is a contractile diaphragm *i*, the **iris**, with a circular orifice (the **pupil**) near to its centre. The **retina** (*r*), which is the portion of the eye's inner surface sensitive to light, is liberally supplied with nerve-fibres and blood-vessels. The nerve-fibres originate from the **optic nerve** (*n*), which enters the eye on the inside of the centre of the retina. Light falling upon these nerve-fibres appears to set up nervous stimuli which are transmitted to the brain, and these are interpreted as the phenomenon termed sight.

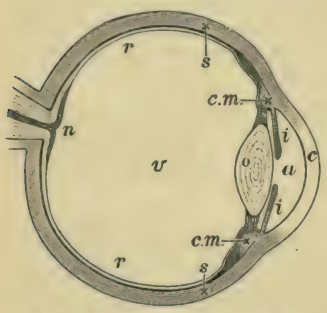

FIG. 209.—Horizontal cross-section through the centre of an eye.

Fig. 210 indicates how the cornea and crystalline lens give rise

to an inverted image of a distant object on the retina Although the image is inverted, yet the mental interpretation of the effect upon the retina is just as though the images were erect.

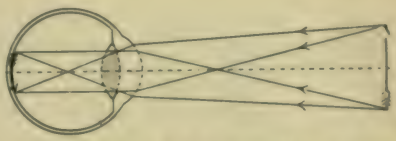

Fig. 210.—Formation of an inverted image on the retina.

It might be thought that a clear image of any object is only obtained when the object is at one given distance from the eye; but there is really a wide range of distance through which distinct vision is possible to a normal eye. This **power of accommodation**, as it is termed, is effected by an involuntary change in the curvature of the surfaces of the crystalline lens. The change in curvature is brought about by the unconscious action of the **ciliary muscle** (*c.m.*, Fig. 209) which surrounds the edge of the lens and is connected to the inner walls of the eye. When a near object is looked at, the ciliary muscle contracts and causes the lens to bulge more, thus increasing its diverging power. On the other hand, when looking at a distant object the ciliary muscle is relaxed, and the lens is thereby flattened. The power of accommodation is limited, for objects which are very near cannot be focussed clearly: nor can the details of very distant objects be seen clearly. The distance at which objects are seen with greatest distinctness by a normal eye varies from 25 to 30 cm.

Defective eyes.—The most common ocular defects are (i) **Short-sight** (or, Myopia), (ii) **Long-sight** (or, Hypermetropia), and (iii) **Loss of accommodative power** (or, Presbyopia).

(i) **Short-sight.**—A short-sighted eye cannot see distant objects distinctly. The eye-ball is usually too long; and parallel rays falling upon the cornea are brought to a focus F (Fig. 211, A) in front of the retina. If the object from which the rays proceed is brought nearer to the eye, the position of the image recedes; and, at a certain distance, vision becomes distinct. If brought still nearer, the power of accommodation enables the lens to thicken, and vision remains distinct. Since the power of accommodation is limited, vision becomes indistinct again if the object is brought too near.

Fig. 211, A, represents how parallel rays may be brought to a focus on the retina by placing a diverging lens L in front of the eye. The dotted lines indicate the path of the rays after passing through the lens. Thus, a short-sighted eye needs a diverging lens in order to see distant objects.

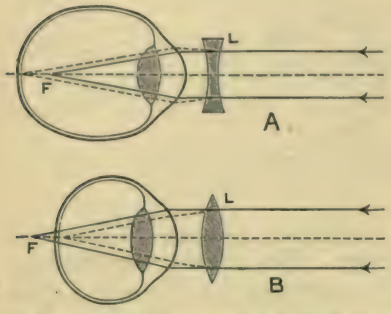

FIG. 211.—In A a double concave lens corrects the short-sighted eye, and in B the double convex lens corrects the long-sighted eye.

(ii) **Long-sight.**—A long-sighted eye cannot see near objects distinctly. When the eye is at rest, parallel rays are focussed behind the retina; but, as a rule, the accommodative power is sufficient to enable distant objects to be seen clearly, though this is not sufficient to focus near objects. The dotted lines in Fig. 211, B, indicate how, by using a converging lens L, the focus of parallel rays may be moved forwards on to the retina of a long-sighted eye.

(iii) **Loss of accommodative power.**—This defect is found usually in elderly people. It is noticed generally that, as the condition develops, the nearest point at which vision is distinct gradually recedes. For this reason, it is often the case that an elderly man finds that printed matter can be read only if held at arm's length.

An object (such as printed matter) held near the eye can only be seen clearly by an eye with this defect if seen through a convex lens, since by this means the image is more distant than the object (Fig. 204, p. 295). On the other hand, distant objects can only be seen with the aid of a concave lens, since the image is brought nearer than the object. Persons suffering from loss of accommodative power find it necessary, therefore, to use two different kinds of lenses, according to the distance of the object viewed.

Near point and far point.—The power of accommodation varies in different individuals, and in the same individual it

changes with progressive age. The normal eye of an adult can focus clearly an object which is not more than 10 or 12 inches distant from the cornea.

The nearest point to the eye at which a small object can be seen clearly is termed the **near point**. The point to which the eye is focussed when at rest is termed the **far point**; and, in the case of a normal eye, the far point is obviously at an infinite distance; but it is frequently found that the far point is within measurable distance of the eye.

The determination of the near point and the far point constitutes an instructive experiment. One method is based upon the fact that when an object is viewed through a convex lens of known focal length and placed quite near to the eye, a well-defined image is seen only when the image is situated between the near and far points. When the object is brought gradually nearer to the lens a point is reached when the image just ceases to be clearly defined; by using the formula on p. 296 the position of the image, and therefore of the near point, can be calculated. Similarly, when the object is withdrawn gradually, a position is reached when the image again ceases to be defined clearly, and the position of the image, obtained by calculation, gives the distance of the far point.

EXPT. 194.—**Determination of near point and far point.** Select a convex lens of known focal length (20 cm. is convenient). Draw a small square, in pencil, on a piece of cardboard, and support this vertically on a movable support behind the lens. Place one eye as near as possible to the lens, and move the cardboard slowly towards the lens until the image is just becoming indistinct; measure this distance. Repeat the measurement twice, and take the average d_1. The distance of the image, *i.e.* the distance of the *near point*, can be calculated by means of the formula $1/f = 1/v - 1/u$. Thus, in an actual experiment, $f = -20$ cm., and $d_1 = 10{\cdot}1$ cm.; hence

$$-\frac{1}{20} = \frac{1}{v} - \frac{1}{10{\cdot}1},$$

or

$$\frac{1}{v} = \frac{1}{10{\cdot}1} - \frac{1}{20},$$

or

$$v = \frac{202}{9{\cdot}9} = 20{\cdot}4 \text{ cm.}$$

Similarly, note the maximum distance, d_2, at which the image is just distinct, and calculate the distance of the far point. Enter the observations thus :

Eye.	d_1.	Near Point (calculated).	d_2.	Far Point (calculated).
Right	6·9	10·5	13·6	42·5
Left	10·1	20·4	14·7	55·5

The visual angle.—The apparent size of an object is proportional to the size of the image of the object upon the retina, and this depends upon the distance of the object from the eye—the nearer the object the larger the image. The apparent size of the object is measured usually by the angle which the object subtends at the eye; and this angle is called the **visual angle.** Thus, in Fig. 212, if E represents the eye and AB the object, the visual angle is AEB; if *ab* represents the object the visual angle is *a*E*b*.

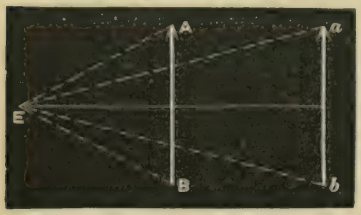

FIG. 212.—Visual angles AEB and *a*E*b*.

The simple magnifying glass.—The apparent size of an object, when viewed by the naked eye, is a maximum when the object is situated at the near point. The apparent size would be increased if the object were brought still nearer, but the image would be indistinct. When, however, the object is viewed through a convex lens held near to the eye, the object may be brought nearer to the eye than the 'near point,' and still the image may be well-defined, since the virtual image due to a convex lens is further from the lens than the object. Fig. 213 represents how a virtual image of an object AB is seen

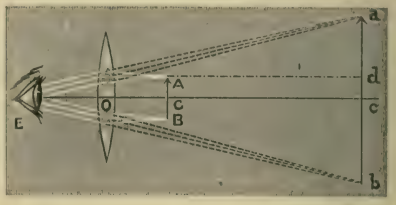

FIG. 213.—The simple magnifying glass.

at ab; and it is evident that the visual angles of the object and of its image are the same. Hence the lens serves the purpose of enabling the visual angle to be increased while maintaining at the same time distinct definition.

In determining the **magnifying power** of a lens it is usual to adjust the position of the object so that the image is situated at the near point; and the magnifying power may be defined as *the ratio of the visual angle of the image to that of the object if it were situated at the 'near point.'*

In Fig. 213 suppose that c is the 'near point,' then

$$\text{magnifying power } (m) = \frac{ac}{dc} = \frac{ac}{AC} = \frac{Oc}{OC}.$$

If Oc be the distance d of the 'near point,' then, since the lens is converging and its focal length therefore negative,

$$-\frac{1}{f} = \frac{1}{d} - \frac{1}{u},$$

and

$$\frac{1}{u} = \frac{1}{d} + \frac{1}{f},$$

or

$$u = \frac{df}{d+f}.$$

Hence,

$$m = \frac{Oc}{OC} = \frac{d}{u} = \frac{d+f}{f}.$$

EXAMPLES.— 1. A long sighted person cannot see objects clearly at a distance of less than 50 cm. Find the power, in dioptres, of the glasses required in order to read print held 15 cm. in front of the glasses.

The focal length must be such that when an object is 15 cm. in front of the glasses, the image is at least 50 cm. distance. Hence, $u = +15$ cm., and $v = +50$ cm.

$$\therefore \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$= \frac{1}{50} - \frac{1}{15} = -\frac{35}{750},$$

or

$$f = -21{\cdot}4 \text{ cm.},$$

or

$$\text{power} = +\frac{1}{0{\cdot}214} = +4{\cdot}67 \text{ dioptres.}$$

2. A person with short-sight is able to read print only when held 15 cm. from the eye. What kind of glasses, and of what focal length, are necessary in order that print held at a distance of 25 cm. from the eye may be read clearly?

In this case, $u = +25$ cm., and $v = +15$ cm.;

$$\therefore \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$= \frac{1}{15} - \frac{1}{25} = +\frac{2}{75},$$

or $$f = +37{\cdot}5 \text{ cm.}$$

Hence, concave glasses of focal length 37·5 cm. are required.

The photographic camera.—The photographic camera consists of a rectangular box with two opposite sides vertical and rigid.

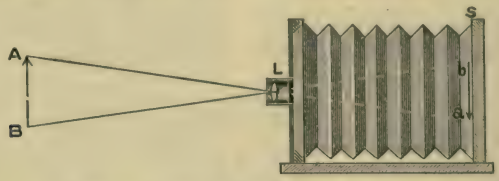

FIG. 214.—Principle of the photographic camera.

The distance between these sides can be adjusted by the remaining sides being made extensible, and consisting of folded leather or other opaque material. A circular opening in one of the rigid sides serves to carry a converging lens (L, Fig. 214); and the opposite side is fitted with a translucent screen S of matt glass, which can be replaced by a photographic plate.

When the camera is placed with its lens directed towards a distant object AB, a real inverted image *ab* is formed on the other side of the lens. The screen S is moved towards the lens until its position coincides with *ab*, when a well-defined image of the object can be seen on the screen. A photographic plate consists of a glass sheet covered on one side with a gelatine film loaded with chemical compounds sensitive to actinic (p. 233) light rays. When such a plate is substituted for the screen, and exposed for a definite period to the rays passing through the lens, subsequent development of the film reveals a permanent reproduction, in black and white, of the distant object. The plate, when developed, is termed a **photographic negative**.

If the upper part of the picture be more distant than the lower part from the lens, then in order that the whole image may be defined with equal clearness in all parts, the lower part of the screen must be placed nearer to the lens than the upper part. For this reason, the frame carrying the screen is frequently hinged along its lower side to the base of the apparatus, so that it may be tilted backwards or forwards according to the relative distances of the upper and lower parts of the picture.

The optical lantern.—The optical lantern consists essentially of a bright source of light and a system of converging lenses

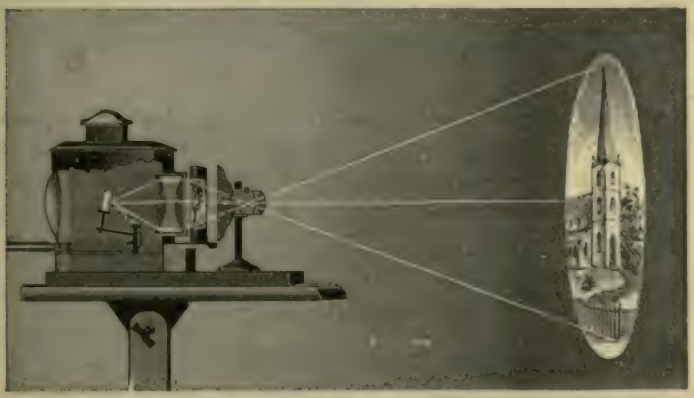

FIG. 215.—The optical lantern.

arranged so as to throw on a screen an enlarged inverted image of a transparent photograph or drawing, or of any object constructed in a manner suitable for projection on a screen. The rays of light proceeding from the source of light (Fig. 215) pass through the transparency (or 'lantern slide') which is supported in a suitable carrier; the rays which are not absorbed by the dark parts of the slide then pass through a **focussing lens**, which is adjusted in position so that the slide is slightly beyond its principal focus, and a real inverted and enlarged image of the slide is thrown on a distant screen. The **condensing lenses** which are usually a combination of two lenses and are nearest the source of light, serve two purposes: (i) to increase the total amount of light concentrated on the slide, and (ii) to deflect the rays of light

passing through the outer portions of the slide sufficiently for their paths to pass through the focussing lens. In the absence of a suitable condenser, the image on the screen would be much less bright, and it would represent the middle part only of the slide.

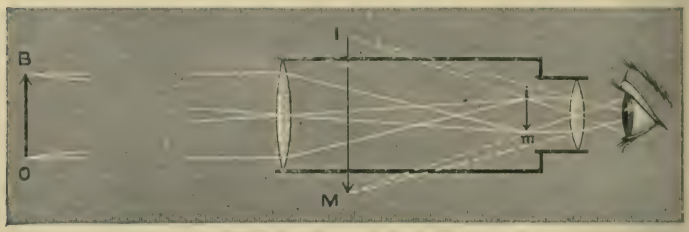

FIG. 216.—Principle of the astronomical telescope.

The astronomical telescope.—In order to obtain a magnified image of a distant object it is necessary to use more than one lens. Fig. 216 represents the principle of the astronomical telescope, which consists usually of a large lens, of considerable

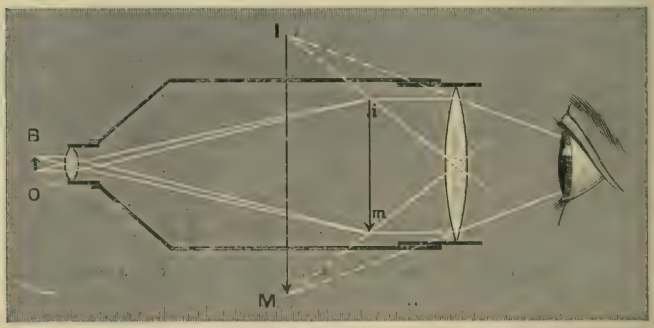

FIG. 217.—Principle of the compound microscope.

focal length, called the **object-glass**, and a smaller lens of short focal length called the **eye-piece**. The diagram represents several rays coming from a point of a distant object, and, since the distance is assumed to be great, these rays are practically parallel. A real, inverted, and diminished image *im* of the object is formed in a position which practically coincides with the focus of the object-glass. The eye-piece is adjusted so that *im* is just within

its principal focus. On looking through the eye-piece a virtual and enlarged image of *im* is seen at IM. The dotted lines in the diagram indicate how the position of this image is determined.

The compound microscope.—In the compound microscope (Fig. 217) a small object-glass of short focal length is used instead of the large object-glass of long focal length employed in the telescope. If the object OB under observation be placed just beyond the principal focus of the object-glass, a magnified, real, and inverted image is obtained at *im*. When this image is viewed through the eye-piece, a virtual image IM of a much larger size is seen.

EXERCISES ON CHAPTER XXIII.

1. A man who can see distinctly at a distance of 1 foot, finds that a certain lens when held close to his eye magnifies small objects 6 times. Determine the focal length of the lens.

2. A man who can see most distinctly at a distance of 5 in. from his eye, wishes to read a notice at a distance of 15 ft. off. What sort of spectacles must he use, and what must be their focal length?

3. A long-sighted person can only see distinctly objects which are at a distance of 48 cm. or more. By how much will he increase his range of distinct vision if he uses convex spectacles of 32 cm. focal length?

4. A long-sighted person uses convex glasses of 40 cm. focal length, and finds that he cannot read print through them comfortably when it is held nearer than 30 cm. What is his nearest point of distinct vision?

5. Describe the simple magnifying glass and explain its use. Draw a figure showing the position of object and image and the course of light when a magnifying glass (focal length = 3 cm.) is used by a person whose distance of most distinct vision is 15 cm.

6. A person, whose distance of most distinct vision is 10 cm., uses a simple magnifying glass to view a certain object. The focal length of the magnifying glass is 2·5 cm. Draw a figure showing the relations between the linear dimensions of object and image, and trace the course of a small pencil of light from a point of the object to the eye.

7. Describe, explaining how the image is formed, either a simple microscope or a simple astronomical telescope.

8. A patient cannot see objects clearly at a distance of less than 36 inches; find the focal length of the glasses required to enable him to read at 10 inches.

9. A short-sighted person cannot see objects clearly at a distance greater than 6 inches. What spectacles would be required to enable him to see distant objects clearly? If his least distance of distinct vision without glasses is 3 inches, what would it be with the above spectacles?

10. Two converging lenses, A and B, are fixed vertically with their axes coinciding and with their centres 70 cm. apart. The focal lengths of A and B are +35 cm. and +20 cm. respectively. A capital letter 10 cm. high, cut from a poster, is fixed vertically 100 cm. from the centre of A and on the distant side from B. The letter is viewed through both lenses by an eye placed near to B. Draw a diagram to scale, showing the position and size of the image of the letter.

11. A reading-glass of 8 cm. focal length is used by a person whose 'near point' is at a distance of 24 cm. What is the magnifying power of the glass?

12. How would you arrange two convex lenses to form a telescope? What is the purpose of each lens?

CHAPTER XXIV.

COMPOSITION OF LIGHT.

Dispersion.—The muiticoloured rainbow, varying from violet at the inner edge to red at the outer, is a familiar phenomenon; and the colour effects seen when the edge of a window frame is viewed through a prism, held with its refracting edge parallel to the object, are also familiar appearances. In this simple experiment we are repeating that historical experiment with which Newton proved that ordinary white light is a complex mixture of coloured lights. Fig. 218 represents the nature of this experiment. A beam of sunlight was caused to fall upon a glass prism held so as to refract the light downwards upon the opposite wall of a dark room. The light was found to be drawn out into a long coloured band, violet at the lower end and red at the upper. This coloured band Newton termed the **spectrum.** The colours violet, indigo, blue, green, yellow, orange, and red are clearly distinguishable. although each colour changes, by insensible

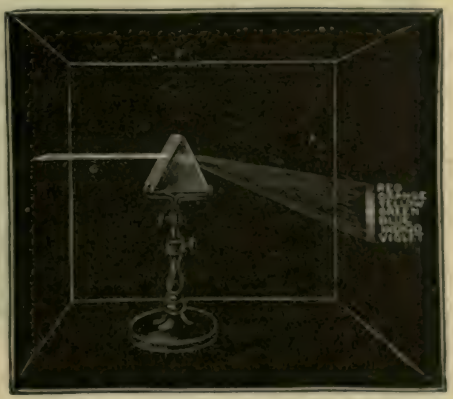

FIG. 218.—Dispersion of light by a prism. (Newton's experiment.)

gradations, into the next. It is evident that, when passed through a prism, violet rays are bent more than yellow rays, and yellow rays more than red rays; or, expressing the same fact in other words, violet rays are more **refrangible**, that is, deflected more from their original path than yellow rays, and yellow more refrangible than red rays. The separation of the different colours, owing to this difference in refrangibility, is termed **dispersion.**

In order to understand why we can obtain a spectrum by this simple means, it is necessary to realize that the condition which gives rise to the sensation of light really consists in the rapid transmission of a kind of wave-motion through the luminiferous ether (p. 233)—a medium which fills all space, even a perfect vacuum, and occupies the intermolecular spaces in all forms of matter. There is reason to believe that when light is being transmitted through the medium, there exists within it a vibratory

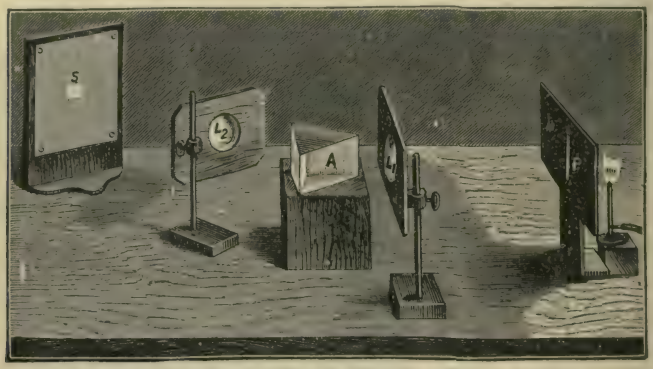

FIG. 219.—Experiment to show the dispersion of light by a prism.

condition, the vibrations taking place at right angles to the direction in which the light is travelling; in this sense light waves may be compared to the ripples on a liquid surface. In just the same way that we can set up either long or short ripples in water, so the waves of light may be of different length; and it is this difference in wave length which determines the colour of the light. The shortest waves to which the eye is sensitive give rise to the sensation of violet, and the longest to that of red. By special means it is possible to detect waves shorter than violet, or longer than red; but they cannot be observed by means of the unaided eye.

EXPT. 195.—**Dispersion by a prism.** In a piece of card cut a slit (P) about 2 cm. long and 1 mm. wide. Place the card, with the slit vertical, in front of a fish-tail gas flame (Fig. 219); and adjust the position of a converging lens (L_1) so that it is approximately at a distance equal to its focal length from the slit. Place a second lens (L_2) a few inches in front of L_1 and so that their axes coincide. Adjust the screen (S) so that a well-defined image of the slit is seen on its surface. Measure the distance between L_2 and S. Arrange a prism (A) on a stand, so that it is of the same height as the slit, and has its refracting edge vertical; and adjust its position so that the rays emerging from L_1 are incident at a suitable angle upon one of its faces. Catch the light emerging from the prism by a second lens (L_2). Move the position of the screen (S) until the coloured band of light is best seen. The distance of the screen from L_2 should be the same as before. Observe that the light is refracted towards the base of the prism, and that it is decomposed into constituent colours, which are differently bent by the prism. The violet light is refracted most and the red light least. Colours between these limits are bent by intermediate amounts. Name the colours you can see.

Analysis of light by a prism.—When a beam of sunlight is made to pass through two prisms similarly arranged instead of one, the coloured band or spectrum produced is longer, that is, the dispersion is greater. The amount of dispersion also depends upon the material of which a prism is made. Glass produces a much greater amount of dispersion than water; flint glass possesses twice the dispersive power of crown glass; carbon bisulphide, again, has even more dispersive power than flint glass.

Although a continuous band of colour is observed when sunlight, or limelight, or a gas- or candle-flame is seen through a prism, this **continuous spectrum** is not always produced. For when substances such as sodium, strontium, and lithium, or their compounds, are burnt in a non-luminous flame, and the light from the coloured flame observed through a prism, a spectrum is seen consisting of bright lines, which are different for different substances. A prism may thus be used, and is used, to analyse light. The light of incandescent sodium vapour, produced by burning common salt in a flame, when observed through a prism is characterised by a yellow line, and the light emitted by other substances when burning are each distinguished by rays of a

particular colour and position in the spectrum. It is thus possible to analyse a substance by examining the light it emits when rendered luminous by suitable means. The instrument used in such investigations is termed a **spectroscope.**

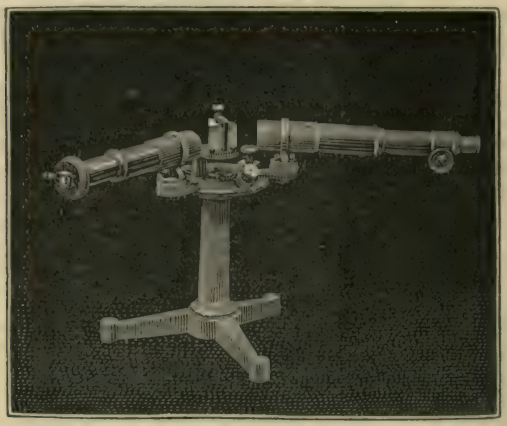

FIG. 220.—A spectroscope.

Spectrum analysis.—A spectroscope (Fig. 220) consists essentially of one or more prisms, with an arrangement known as a slit for limiting the breadth of the beam and a convex lens for making the rays parallel, and a small telescope for viewing the analysed light. When such an instrument is fitted upon a telescope and the telescope is directed towards the sun, a rainbow-coloured band having numerous dark lines at right angles to its length is observed. These lines are the representatives of substances the luminous vapours of which exist in the sun, and by identifying them with lines produced by burning terrestrial substances, the materials of which the sun is composed have been found. The same principle applies to the stars or other celestial bodies.

Formation of white light from its constituents.—Just as it is possible to analyse white light, splitting it up into its constituent colours or wave-lengths, so, by suitable arrangements, these separated or dispersed colours can be made to recombine, forming white light over again. This building up, or synthesis, of white light can be effected in the following ways:

1. By interposing a second prism of the same material and angle as the first, with its angle reversed. The dispersion of the first prism is neutralised, and the beam of light leaves the second prism in a direction parallel to the beam incident upon the first prism.

2. By the colour disc.

EXPT. 196.—**Recomposition of light by means of a second prism.**—Form a spectrum as described in Expt. 195. Place a second prism (B, Fig. 221), of the same material and having the same refracting

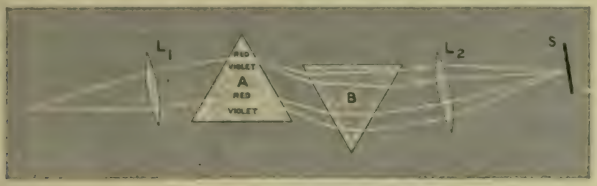

FIG. 221.—Recomposition of light.

angle as prism A, in the position shown in the diagram, adjusting the second prism so that the rays fall upon one of its faces, and so that its faces are parallel to those of the first prism. Place the lens (L_2) and the screen (S) in the same relative position as before, and observe on the screen the *white* image of the source of light. Evidently the different colours of the spectrum have been recombined to form white light.

It is an important fact that, by the simple laws of refraction, the path of each of the rays through B must be parallel to its path through A; also, since the rays emerging from B must be parallel to the rays incident upon the prism A, the former rays must constitute a parallel beam, and they will be brought to a focus at the principal focus of the lens L_2, and a white image therefore is formed.

EXPT. 197.—**Recomposition by colour disc.** Upon a round piece of card paint sectors of the different colours contained in the spectrum, arranging the areas of the coloured sectors as nearly as possible in the proportion in which they occur in the spectrum.

Place the card upon a whirling table or upon a top, and rotate it rapidly, when it will be found that light from the card gives rise to the sensation of an impure white or grey.

The colour disc.—The explanation of the recombination of the separate colours of the spectrum by means of a rapidly revolving disc, as in Expt. 197, is very simple. It is due to

what is called the **persistence of images** on the retina of the eye. Each impression the retina receives lasts for a certain length of time—about one-tenth of a second. It is not an instantaneous impression only. Think of the common trick of whirling round a stick with a spark on the end which gives rise to the impression of a continuous circle of light. This is because the second impression of the spark is received by the eye before the first impression has died away. Similarly, the impression of one sector, say, a red one, has not disappeared before the next is received, and while these compounded impressions linger a third one comes along. The blurred total of all these rapidly occurring impressions produces the greyish white tinge seen when a colour disc is whirled.

Rainbows.—Rainbows are caused by sunlight falling upon drops of water whether in the form of rain or of spray. The observer must have his back to the sun; and the centre of the bow is the point in the sky directly opposite to that occupied by the sun at the time of observation. In the **primary rainbow** the red colour is on the outer edge and the violet on the inner edge, the other colours of the spectrum being between them: in the **secondary rainbow** sometimes seen above the primary one this order of colours is reversed, the red being on the inner edge and the violet on the outer. Suppose the sun to be on the horizon when an observer sees these rainbows. The centre of the bow would be on the opposite point of the horizon, the red top of the primary rainbow would be at an angle of about 42° above the horizon, and the violet edge would be about 2° below the red: the angular elevation of the secondary rainbow would be about 52°. The radius of the primary rainbow as a whole is always about 41° and that of the secondary bow about 52°. When, therefore, the sun is on the horizon the bows seen are the largest possible. As the position of the sun above the horizon increases, the centre of the bows gets more and more below the horizon, the arcs visible become smaller and smaller until, when the sun has an altitude of about 41°, the primary bow disappears; while the secondary bow also becomes invisible when the sun's altitude exceeds 52°. This explains why rainbows are never seen in the British Isles in the middle of the day in summer.

The optical cause of the rainbow is a little difficult to explain exactly in an elementary book, but Fig. 222 may enable the student to comprehend the general principle involved. Suppose the sun's rays to be falling in the direction indicated upon the raindrops A_1, A_2, A_3, A_4. The light falling upon A_1 passes into the raindrop, is internally reflected along the path *abc* and emerges dispersed into the spectrum colours from violet *v* to red *r*. Light entering the second drop A_2 is similarly affected. From the lower drop violet light reaches the observer's eye; and red light from the upper drop, while the various drops which may be imagined between these two contribute the intervening colours of the spectrum according to their positions. In the case of the secondary rainbow, the sun's rays undergo double internal reflection in the raindrops, as shown in A_3 and A_4. Light enters A_3, and after traversing the path *mopq* emerges, broken up into the spectrum colours from red to violet in the order indicated in the figure. Red light thus reaches the observer's eye from A_3 and violet light from A_4; and the intermediate colours are formed in similar manner.

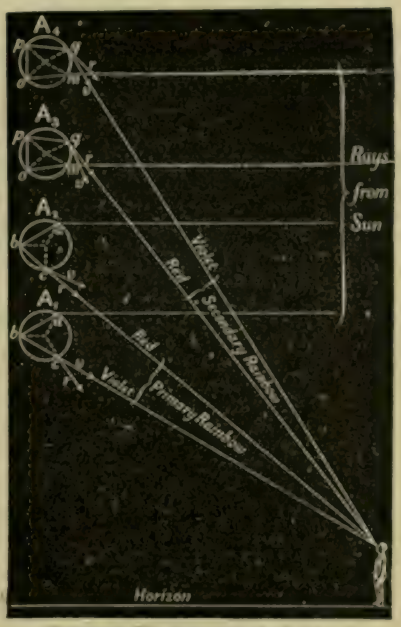

FIG. 222.—The formation of a rainbow.
Adapted from the *U.S. Monthly Weather Review.*

COLOUR.

Colour of transparent bodies.—The colour of transparent bodies is due to the constituents of white light transmitted by them. A blue solution through which the light from a lantern is passed is blue because, of all the colours of the spectrum, it is able to transmit easily only the blue rays; the others—green, yellow, orange, red, etc.—are absorbed by the solution. Consequently, if this trans-

mitted blue light falls upon a sheet of red glass it is, in its turn, absorbed; red glass only transmits red light, that is why it is red. So that a combination of the blue solution and a piece of red glass is quite opaque to light—none of the colours of the spectrum can pass. Similarly, pieces of red and blue glass together are, if thick enough, quite opaque. When a strip of coloured glass, or a solution in a narrow test tube, is held between a spectrum and a screen, it appears as a black shadow upon the screen in all parts of the spectrum except in the colour which it is able to transmit. Transparent bodies like glass, water, and so on, transmit all the colours of the spectrum with nearly equal facility, and therefore appear colourless when only thin layers are used. As, however, no substance transmits light of all wave-lengths equally, there is no perfectly transparent body.

EXPT. 198.—**Absorbed and transmitted rays.** (i) Make an Oxford blue liquid by adding ammonia solution to copper sulphate dissolved in water until the precipitate formed is re-dissolved. Place the blue liquid thus made in a glass cell. Focus the round hole of a lantern cap on the screen. Interpose the filled cell. Notice the pure blue colour. Now interpose red glass either before or behind the cell. Notice that no light can pass now.

(ii) Use a hollow prism containing carbon bi-sulphide to produce a long spectrum of a slit on the lantern cap. Pass the filled glass cell from the last experiment through the spectrum, and notice that it is only able to transmit blue light. Repeat the experiment with as many coloured transparent solutions as possible, *e.g.* a solution of bichromate of potash and of permanganate of potash. Notice in each case that a particular liquid is only able to transmit light of its own colour.

Colour of opaque bodies.—The colour of opaque bodies is due to the constituents of white light which they reflect. If the light from a lantern in an otherwise dark room be made to fall upon sheets of cardboard which have been painted with various brilliant colours, and the light reflected from the coloured sheets be caught on a white surface, it is at once seen that the colour of the light reflected is the same as that of the card from which it comes.

Coloured opaque bodies when passed through a spectrum only appear coloured when in that part of the spectrum which is the colour they appear to have in white light. A red substance like sealing-wax is red only when there are red rays falling upon it

which it can reflect. The sealing-wax absorbs all the other constituents of white light; and hence when it is held in blue light, or light of any other colour than red, since all the light rays of this colour are absorbed, no light is reflected and it appears black. A white opaque substance, like a sheet of paper, appears white because it reflects all the constituents of white light equally well. Similarly, when a card painted violet is passed through a spectrum it only appears violet when in the violet rays, and in all other colours it seems black, because it cannot reflect these colours.

EXPT. 199.—**Absorbed and reflected rays.** (i) Paint sheets of cardboard with various brilliant colours. Send the light from a lantern in an otherwise dark room upon them, and catch the reflected light on white sheets of cardboard. Notice that the colour of the light reflected is the same as that of the card from which it is reflected.

(ii) Pass through the same spectrum various coloured opaque bodies, *e.g.* a rod of sealing-wax. Notice in this case it is only coloured when passing through the red rays. It appears a dull grey colour in most parts of the spectrum. Observe that green leaves are only coloured when passing through the green part of the spectrum.

Selective absorption and transmission.—In every case the colour of a body depends on selective absorption or selective transmission. Of the coloured rays of white light one portion is absorbed at the surface of the body. The body is **coloured and transparent** if the unabsorbed portion traverses it; if, on the contrary, the unabsorbed light is reflected the body is **coloured and opaque**. In both cases the colour depends upon the constituents of white light which are left to reach the eye after the other constituents have been absorbed. Bodies which reflect or transmit all colours in the proportion in which they exist in the spectrum are white; those which reflect or transmit none are black. Between these extreme limits infinite tints exist depending on the smaller or greater extent to which bodies reflect or transmit some colours and absorb others.

Bodies have no colour of their own; the colour of a body changes with the light which falls upon it. It is interesting to remember that this absorption of certain constituents of light necessitates a using up of energy. But since energy cannot be destroyed it is in these cases converted into heat. Theoretically,

a blue glass would get hotter than a red one, because the former absorbs all the red rays, and these have a greater heating effect than blue rays.

EXERCISES ON CHAPTER XXIV.

1. Describe and explain the effects observed when cards coloured bright red, green, and blue respectively, are passed from the red to the blue end of the spectrum.

2. Some glass houses in which ferns are grown are constructed of green glass. Describe the appearance, to an observer in such a house, of a lady in a red costume carrying a book with a bright blue cover. Give reasons for your answer.

3. How would you explain to a class of children the effect of a stained glass window upon sunlight? What simple experiments would you perform to convince them of the truth of your statements?

4. A ray of white light is passed through a glass prism; make a sketch showing how the direction of the ray is changed by its passage through the prism and the order of the colours seen when the light falls on a screen.

How would you show that when these colours are re-combined white light is produced?

5. Describe an arrangement by means of which a spectrum may be formed upon a screen.

If the light is made to fall upon a piece of red glass before reaching the screen, how and why will the spectrum be affected? What would the effect have been if blue glass had been used?

6. How can it be proved that:

(*a*) White light is a mixture of many colours? (*b*) Different colours have different degrees of refrangibility?

7. What is meant by the dispersion of light? On what fact does it depend?

8. Explain the term refrangibility as applied to a ray of light. Are rays of all colours equally refrangible?

9. It is sometimes said that "red glass colours the sunlight red," and that "blue glass colours the sunlight blue." Mention facts or experiments which show that this is not accurate. Put the statement in a more accurate form.

10. Bright sunlight falls obliquely upon the surface of the water contained in a white china basin; a penny is held near the surface of the water, and in such a position that its shadow falls upon the bottom of the basin. Parts of the shadow are found to be edged with colour. What colours may be observed? On what part of the shadow is each to be seen? How do you account for the colours?

11. What is a prism? Give a diagram showing the course of a ray of white light through a prism made of glass. Which colour is refracted most, and which least?

12. Describe the essential parts of either (*a*) a spectroscope, or (*b*) an astronomical telescope.

13. Explain the coloured fringes seen when objects are viewed through a glass lustre or prism.

14. Why does a field poppy appear red? What experiment could you arrange to make it appear black?

15. Several coloured cards (*e.g.* red, blue, green) are lying on a table in a lighted room. State how far the colours could be identified by a person seeing them through a piece of cobalt (blue) glass. Give reasons for your answer.

16. Light from a slit is allowed to fall on a prism. State and explain what may be observed when the slit is illuminated with (i) sodium flame, (ii) white light.

17. Describe a spectroscope. What will be observed in the instrument when the light passing through the slit comes from (i) a spirit lamp with a salted wick, (ii) an electric light, (iii) an electric light in a red glass globe?

18. How would you test, in a given instance, whether the light transmitted by a blue glass is homogeneous or not?

PART V.

SOUND.

CHAPTER XXV.

VIBRATORY MOTION.

Wave motion.—The circular waves set up when a stone is dropped into still water is a phenomenon familiar to all. Although the waves appear to travel outwards, yet the water itself is not travelling so; for, if a row of corks be floating on the water the corks move *up and down* only, and their distance from the centre of the waves is not increased: in fact, it is the 'disturbance' alone which travels outwards. This is an example of true wave motion, which can be defined in the following terms: **Wave motion consists in the repeated motion of a series of particles, the motion being handed on from each particle to its neighbour.**

When the stone touches the water, it causes a depression; and, like all elastic bodies which, when deformed, tend to resume their original shape, the water tends to recover its original level. Consequently, water flows into the depression: but, in so doing, the property of **inertia** (p. 103) causes the inflowing water to 'overshoot the mark,' and the depression is followed by an elevation. This is again followed by a depression, and so on. But the disturbance is not restricted to this one point; for, through the agency of the property termed cohesion (p. 57), the disturbance is handed on to neighbouring particles. In fact, the condition resembles what might be observed with a row of pendulums, the bobs of which are joined by spiral springs. A transverse motion to and fro of the first

pendulum will influence the next pendulum, imparting to it a similar motion. This will be handed on to each pendulum in turn, resulting in a kind of wave motion along the row. In this case the persistence of the motion is due to the inertia of each pendulum, and its transference along the row is due to the elasticity of the springs connecting the bobs. Subsequent paragraphs will show how a wave motion, of one type or another, may be propagated through any medium possessing these properties of inertia and elasticity.

Simple harmonic motion.—Suppose a simple pendulum to be set swinging so that its bob describes a circular path: it will traverse the path with uniform velocity. If looked at downwards in a sloping direction, the path will appear to be an ellipse (Fig. 223); but if the eye be placed at the same horizontal level it will appear to move along the straight line A*bcd*.... Suppose the circular path to be divided into 16 equal portions, AB, BC, etc.; then when viewed horizontally, the distances A*b*, *bc*, *cd*, etc., will be the apparent distances traversed in equal time intervals. At both A and I the bob will appear to be momentarily at rest: and its velocity will appear to be a maximum at *e*. This apparent motion along the path A*bcd* is termed **Simple Harmonic Motion** (S.H.M.).

FIG. 223.—A simple pendulum, describing a circular path.

The total time-interval occupied in passing from its position of rest, at *e*, until it again passes the same point in the same direction is termed the **period** of the vibrating body. The **phase** of the vibrating body at any instant is the fraction of a total period which has elapsed since passing the position of rest in a given direction, *e.g.* from left to right. Thus, the phase when passing *f* from right to left will be 7/16. The **amplitude** is the extreme distance to which the body moves from its position of rest; thus, in Fig. 223, the amplitude is *e*A.

Transverse wave motion.—In Fig. 224 (i), let A, B, C, ... represent the path of a particle moving up and down with S.H.M. The positions A, B, C, etc., occupied after equal intervals of time, are located by means of the generating circle,

the circumference of which is divided in this case into 12 equal parts.

Suppose a similar motion to be set up in a number of particles, 1, 2, 3, 4, etc., arranged along a straight line at equal distances apart; and suppose the motion to be handed on from one particle to the next. Let the amplitude of each particle be equal to AD, and let consecutive particles differ in phase by $\frac{1}{12}$ of a complete period.

If particle 1 be moving downwards through its position of rest, particle 2 will then be moving downwards at *a*; particle

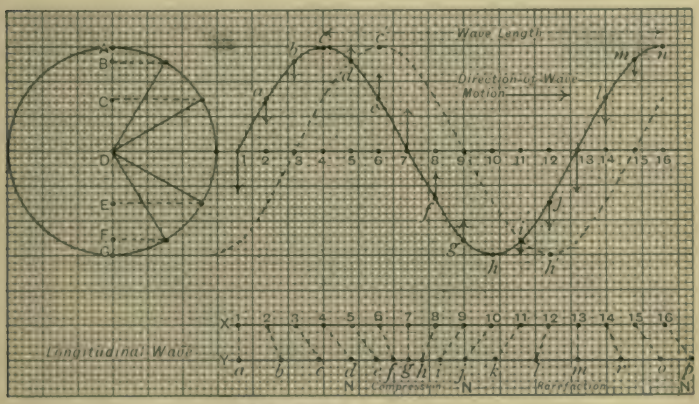

FIG. 224.—Wave motion.

4 will be momentarily at rest at *c*; particles 5 and 6 will be moving upwards at *d* and *e* respectively; 7 will be passing its position of rest in an upward direction, and so on. If these instantaneous positions are joined by a continuous line, a wave outline is obtained which closely resembles the outline of a water wave.

The dotted outline indicates the instantaneous positions occupied by the particles at a later moment, viz., $\frac{1}{6}$ of a period later. The elevation at *c* has now moved forward to the position *c'*; and, after one complete period has elapsed, the elevation at *c* will have travelled to the point *n*. Thus, the wave motion is propagated onwards; and, since the particles vibrate

transversely to the direction in which the waves are moving, the effect is termed a **transverse wave motion.**

The **wave length** is defined as the least distance between any two particles which are in the same phase; thus, the distances *cn*, *bm*, or *al* are equal in each case to the wave length.

The **velocity** of the wave motion is the distance through which the disturbance is propagated in unit time. The **frequency** is the number of complete waves which pass a given fixed point in unit time.

Suppose that n wave crests pass any fixed point in unit time, and that the wave length is l. Then, if V be the velocity of the wave motion, the first crest will have travelled in unit time to a distance V; hence

$$V = n \times l,$$

or velocity = frequency × wave length.

Transverse waves cannot be transmitted through gases, for the elasticity of a gas is brought into operation only when the gas is compressed or rarefied. Suppose, then, that the consecutive particles of a gas are set in vibration to and fro along, instead of transversely to, the line of disturbance. The distances apart of consecutive particles will vary—at one instant the particles being closer together than normally, and further apart at another instant. Such waves are termed **longitudinal waves**; and it may be proved that the transmission of sound through gases involves this type of wave only. Sound is transmitted also through solids and liquids by means of the same type.

Longitudinal wave motion. In Fig. 224 (ii) a row of equidistant particles 1, 2, 3, ... are arranged along the straight line X. Suppose that the particles are set in S.H.M. horizontally, and that the phase difference between consecutive particles is equal to $\frac{1}{6}$ of a complete period. The wave curve *abcd*... of Fig. 224 (i) may be regarded as a *displacement curve* for determining the instantaneous positions of the particles: an upward displacement in Fig. 224 (i) corresponding to a displacement forwards in Fig. 224 (ii), and a displacement downwards to a displacement backwards. Thus, the instantaneous positions of the particles will be represented by the points *abcd*... along the line Y. It is evident that, at the instant represented, the

particles 5 to 9 are closer together, and the particles 11 to 15 are further apart, than normal; or, in other words, these are regions of compression and of rarefaction respectively. Also, the pressures in the neighbourhoods of particles 4 and 10 must be normal. A longitudinal wave may be considered, therefore, to consist of a sequence of compressions and rarefactions separated by regions of normal pressure; and this condition is transmitted onwards with definite velocity.

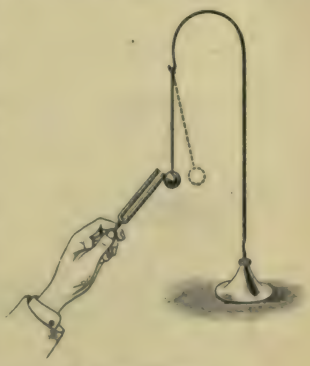

FIG. 225.—Expt. 200.

The production of sound. The origin of all sound is motion. Thus, when sound is derived from a stretched string, its indistinct outline shows that it is vibrating rapidly to and fro.

EXPT. 200.—**Vibration and sound.** Strike the end of one prong of a tuning-fork on the knee, or on a hard cushion. Hold the fork so that the outer side of one of the prongs touches either (i) the lip, or (ii) a pith ball suspended by thread (Fig. 225) or (iii) the surface of water contained in a beaker. The effect, in either case, demonstrates that the prongs are vibrating to and fro.

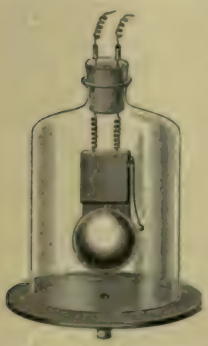

FIG. 226.—Experiment to show that sound is not transmitted through a vacuum.

The transmission of sound.—The fact that some definite medium is necessary for the transmission of sound may be demonstrated by means of the apparatus shown in Fig. 226. An electric bell is suspended by wire springs inside the bell jar of an air-pump. The bell is connected electrically to a voltaic cell by means of wires passing through the rubber stopper of the bell jar. When the jar is full of air the ringing of the bell can be heard distinctly; but, as the air is exhausted, the sound becomes almost inaudible.

Solid media, also, may serve for the transmission of sound: thus, if a watch be placed on one end of a table (or, in contact with one end of a long rod) the ticking is heard distinctly by an ear placed in contact with the other end of the table or rod.

The generation of sound waves by a vibrating body.—In Fig. 227 (i), the letters *a*, *b*, and *c* represent the successive positions of one prong of a vibrating tuning fork. The movement of the prong resembles closely that of a simple pendulum: when in position *a* the prong is momentarily at rest and is commencing to move towards the right; its velocity increases until it is a maximum at *b*, and then diminishes until it reaches *c*, when it is again momentarily at rest. Moreover, the movements are **isochronous**, that is, they are performed in the same time, whether they are small or large.

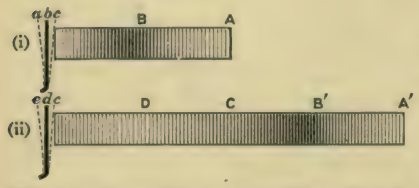

FIG. 227.—Compression wave caused by a vibrating prong of a tuning-fork.

Consider the effect of this movement upon a column of air situated to the right of the prong. The first movement from position *a* causes in the air a slight compression which is propagated outwards with the velocity of sound; subsequent movement of the prong through position *b* to position *c* causes similar compression pulses, but the compression will be most pronounced at the moment when the prong has maximum velocity, *i.e.* when it is passing position *b*. When the prong reaches position *c*, the condition of the air column will resemble that shown in Fig. 227 (i); the first compression will have travelled to a position A, and the pulse of maximum compression will be situated at B. Fig. 227 (ii) represents the subsequent condition of the air column when the prong has moved back again through position *d*, to its original position *e*, thus completing one vibration. The movement of the prong from right to left tends to leave a partial vacuum behind it, and the air is rarefied partially; this rarefaction is a maximum when the prong is moving with maximum velocity through the position *d*.

At the moment when the prong has completed one vibration, the air column will be in the condition represented. The first compression A will have travelled outwards to A′; the region of maximum condensation B will have moved to B′. The region at C will be momentarily in its normal condition; and the region of maximum rarefaction will be at D. The condition thus set up is one complete **sound wave**. After the second vibration of the prong, the first disturbance will have travelled away to a distance twice as far away as A′, and the space between A′ and the fork will be again in the same condition of condensation and rarefaction as shown in Fig. 227 (ii).

It must be borne in mind that the sound waves are not restricted to narrow columns of air, as shown in Fig. 227; in fact, the surrounding air in almost all directions is affected in a similar manner. This would be realised more fully if a point source of sound waves could be considered; and, in such a case, we could regard the point as being surrounded with spherical envelopes of compression and rarefaction spreading rapidly outwards with the velocity of sound.

The **loudness** or **intensity** of the sound depends simply upon the energy contained in that portion of the waves which strike the drum of the ear. The energy contained in any single wave remains practically constant; and, since each wave is distributed over a spherical surface which is enlarging rapidly, the energy passing through each unit area of a wave surface depends upon the distance of that surface from the source.

Since the area of a sphere varies directly as the square of the radius, the energy of the pulse when at a distance of 2 metres from the source will be distributed over an area four times as great as when the pulse was at a distance of 1 metre; and an ear placed at the former distance will experience the effect of only one-fourth the energy which it would experience when placed at the latter distance. Hence, **the loudness of the sound varies inversely as the square of the distance from the source.**

Reflection of sound.—Sound waves are reflected according to the same laws which govern the reflection of waves of light by plane or spherical mirrors; but the conditions under which these two phenomena may be observed differ on account of the wide

difference between the length of light waves and the length of sound waves and also because light waves are transmitted by the ether, whereas sound waves require a material medium for their transmission. The wave length of the lowest audible note is about 36 ft., and that of the highest audible note is about half an inch: these lengths are very great in comparison with the wave length of light. In order that the phenomenon of reflection may be observed, the surface must be large in comparison with the wave length of the vibrations falling upon it. Hence, for the reflection of sound, a surface of considerable area is required. On the other hand, long waves do not require the surface to be so smooth. Consequently, a comparatively rough surface, such as a sheet of cardboard, a wooden board, or a brick wall, serve to produce the phenomenon of reflection of sound.

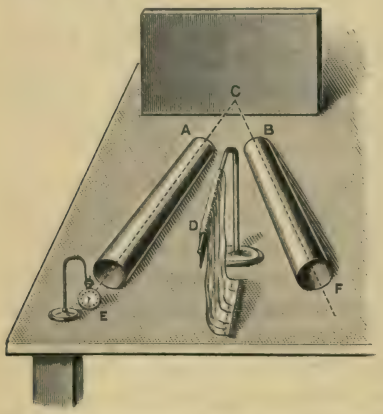

FIG. 228.—Experiment on the reflection of sound.

EXPT. 201.—**Reflection of sound.** A and B (Fig. 228) are two tin-plate tubes, about 1 yard long and 3 inches diameter. Support them horizontally in the positions shown. Suspend a watch at E near to the end of one tube. Place the ear near to the other end of the tube at F. If the sound can be heard, hang up a screen (such as a wet towel) at D so as to prevent the sound waves from passing directly to the ear. Now place a flat reflector in a vertical position at C, and slowly rotate it round a vertical axis. Notice the position of the reflector when the sound can be heard; and observe also the effect of tilting the reflector forwards or backwards.

Echoes are familiar examples of the reflection of sound, and they can be observed frequently in the neighbourhood of houses, high walls, cliffs, or hillsides. If the sound be generated near the observer, it is necessary that the reflecting surface be not less than a certain distance away, otherwise the echo is confused

with the original sound, owing to the fact that the impression on the ear of a sound wave persists for at least $\frac{1}{10}$ of a second. The reflecting surface must be therefore at such a distance that the sound wave will require at least this interval of time to travel to the surface and back again.

A peculiar phenomenon of echo can be observed occasionally when a single wave-pulse of sound is reflected from a series of receding steps, as in a staircase, or from a series of separate flat wooden posts along a roadside. The greater distance of consecutive reflecting surfaces causes consecutive echoes to be retarded slightly in reaching the ear. When the interval between the echoes is regular, the sequence will give rise to a note of which the pitch corresponds to the frequency of the echoes.

Masses of water vapour, such as clouds, or of any gas denser than the air, will serve as surfaces for the reflection of sound. Thus, the irregular roll of thunder which often follows a flash of lightning consists in the numerous and overlapping echoes due to direct reflection from cloud surfaces at different distances and to multiple reflection of the waves between two or more cloud surfaces, or between the clouds and the earth. The original noise is extremely brief, and its duration corresponds with that of the lightning flash.

When a source of sound, such as a watch, is placed at the focus of a concave reflecting surface, the sound waves are reflected along parallel paths, and they can be detected at a greater distance than if the 'sound-mirror' is not used. In the **speaking-tube** the waves are reflected repeatedly from side to side of the tube; and, instead of the energy of the sound waves being distributed through a rapidly increasing space, it remains more or less concentrated within the limits of the tube, and sufficiently so for the sounds to be detected by an ear placed at the distant end.

EXERCISES ON CHAPTER XXV.

1. A regular series of waves is transmitted along a flexible cord. What do you understand by the wave-length and the amplitude of the waves? Would you call the waves longitudinal or transverse? Explain the meaning of these terms.

2. An observer on the sea-shore notes that the waves are breaking at the rate of 10 per minute, and that they take two minutes to reach the shore from a rock 100 yards out at sea. Find the mean wave-length and the velocity of propagation in feet per second.

3. What is meant by saying that the vibration of a pendulum or tuning-fork is isochronous? What would be the effect observed after striking a tuning-fork if the frequency of the vibration increased in the same proportion as the amplitude diminished?

4. Explain the formation of echoes. How is it that the sharp crack accompanying a flash of lightning produces the long roll of the thunder?

5. Two observers are stationed at a distance of 1 mile and $\frac{1}{2}$ mile respectively from a point where a bugle note is sounded. If there are no reflections of the sound, how much louder will the sound appear to the second observer than to the first?

6. Describe an experiment showing that air or some other medium is necessary for the transmission of sound. What practical difficulty arises in such an experiment?

7. Describe an experiment to show that sound is not propagated through a vacuum.

How does the air move when sound waves travel through it?

8. Explain the formation of echoes. A person standing 92 yards from the foot of a cliff hears the echo half a second after clapping his hands. Deduce the velocity of sound in air.

9. Show what must be the direction of sound vibrations, relative to the direction of propagation, in order that sound may be transmitted by a gas. What characteristics of the vibrations determine the character of the sound heard?

10. Describe the action of a vibrating tuning-fork.

11. Describe two experiments which show that sound waves can be reflected. A sharp tap is sounded in front of a long flight of stairs. What impression would you get if you were standing in front of the stairs?

CHAPTER XXVI.

ELASTICITY. VELOCITY OF SOUND.

Elasticity.—The influence of elasticity upon the transmission of wave motion has been referred to previously (p. 328). All forms of matter, when subjected to external forces, undergo changes of volume or of shape; and when the forces are removed, the matter tends to recover more or less completely its original volume or shape. This power of recovery is termed **elasticity.** Thus, a bent clock-spring or a stretched steel wire are examples of bodies which possess elasticity to a high degree. Liquids and gases offer resistance to change of volume only, and not to change of form; and we may say that such substances possess considerable *volume-elasticity.*

The change of volume, or of form, which a substance undergoes, is termed the **strain**; and the force causing it is termed the **stress.** The ratio **stress/strain** is termed the **coefficient of elasticity.**

Elasticity of gases.—When a given volume of a gas, confined under an observed pressure, is subjected to an increased pressure, the volume is diminished by a definite amount, in accordance with Boyle's Law (p. 83). Thus, suppose v c.c. of a gas be measured at a pressure p dynes/cm.^{2}; and let the volume be reduced to $(v - dv)$ c.c. when the pressure is increased to $(p + dp)$ dynes/cm.^{2}, where dv and dp represent small changes in volume and pressure respectively. Then, since the strain is measured by the change in volume of each unit volume of gas, it is represented by the ratio dv/v; and the stress producing this strain is dp dynes. Hence,

$$\text{coefficient of volume-elasticity} = \frac{dp}{dv/v} = \frac{v \cdot dp}{dv}.$$

It can be shown that, in the case of a gas, *this coefficient is numerically equal to the original pressure.* For, by Boyle's Law,

$$pv = (p + dp)(v - dv)$$
$$= pv - p\,.\,dv + v\,.\,dp - dp\,.\,dv.$$

But, since both dp and dv are very small, their product may be neglected;

$$\therefore \quad p\,.\,dv = v\,.\,dp$$

or $$\frac{v\,.\,dp}{dv} = p.$$

Velocity of sound in gases.—Steam is seen issuing from the whistle of a distant locomotive sooner than the sound is heard: the flash of a gun, or the striking of a cricket ball with the bat, is seen before the sound reaches the observer: so also the lightning flash precedes the thunder. Such observations demonstrate that time is required for the transmission of sound from one point to another.

It was proved theoretically by Sir Isaac Newton that the velocity (V) of sound in a gas varies directly as the square root of the volume-elasticity (E) of the gas, and inversely as the square root of the density (D). Or, expressed as an equation,

$$V = \sqrt{\frac{E}{D}}.$$

For example, the density of air, at 0° C. and at a pressure of 76 cm. of mercury, is 0·001293 gm./c.c.; and, by the previous paragraph, E is measured by the pressure acting upon it. The pressure on each sq. cm., due to a column of mercury 76 cm. high, is $(76 \times 13{\cdot}6 \times 981)$ dynes. Therefore

$$V = \sqrt{(76 \times 13{\cdot}6 \times 981)/0{\cdot}001293} = 28000 \text{ cm./sec.}$$

This value is not in agreement with that obtained by actual experiment, viz. 33180 cm./sec.; and the cause of the discrepancy was not explained until a much later date, when it was proved that the numerator of the above equation should be increased by the product 1·41. In this case,

$$V = \sqrt{(1{\cdot}41 \times E)/D} = 33170 \text{ cm./sec.}$$

Experimental determination of velocity of sound in air.—The first attempt to measure the velocity of sound in air was made in 1738 under the auspices of the French Academy of Sciences. Two groups of observers, with cannon, were stationed on hills

about 17 miles apart; one of the cannon was discharged, and the observers on the distant hill measured the time interval between the flash of the cannon and the sound of the discharge. The observation was made in both directions so as to eliminate the effect of wind. From the observations it was calculated that the velocity at 0° C. is about 332 metres per second. Taking these and more recent experiments into consideration, the mean value deduced for the velocity at 0° C. is 331·7 metres (or 1088 ft.) per sec.

Effect of various conditions on the velocity of sound.—Since, by Boyle's Law, the volume occupied by a given mass of gas is inversely proportional to the pressure, it follows that the *density of a gas must be directly proportional to the pressure.* The elasticity of a gas is also directly proportional to the pressure. A change of pressure thus affects equally both the density and the pressure of a gas. The relation of numerator to denominator in the ratio *elasticity/density* is therefore unaltered by any change in the pressure. Hence, the velocity of sound is the same at any atmospheric pressure. This conclusion has been verified by experimental determinations of the velocity at high altitudes.

An increase in temperature reduces the density of a gas. Hence, the velocity of sound increases with rise of temperature, and diminishes with fall of temperature. Based upon the known rate at which a gas expands (p. 164), and upon the fact that the density of a gas is inversely proportional to the volume occupied by a given mass of the gas, it can be proved readily that $V_t = V_0\sqrt{(1+\alpha t)}$, where V_t and V_0 are the velocity of sound at $t°$ C. and 0° C., and α is the coefficient of expansion of a gas (viz. 0·00368). From this equation the velocity of sound at any temperature $t°$ C. is

$(33170 + 61t)$ cm. per sec.,

or $(1088 + 2t)$ ft. per sec.

Damp air is a mixture of ordinary dry air and water vapour. Since the density of water vapour, at ordinary temperatures, is less than that of dry air in the ratio 0·62 : 1, the density of damp air is less than that of dry air at the same temperature and pressure. Hence the velocity of sound in damp air must be greater than in dry air.

As the velocity varies inversely as the square root of the density, other conditions being the same, its relative value in any two gases can be obtained from the densities of the gases. For example, air has a density of 14·3 compared with hydrogen, hence

$$\frac{\text{velocity of sound in hydrogen}}{\text{velocity of sound in air}} = \sqrt{\frac{14\cdot3}{1}}$$
$$= 3\cdot8.$$

Taking the velocity of sound in air to be 1088 feet per second, that of hydrogen is therefore $1088 \times 3\cdot8$ or 4134 feet per second. Similarly, as oxygen is 16 times denser than hydrogen, and $\sqrt{16} = 4$, the velocity of sound in oxygen is $\frac{1}{4}$ of 4134, or 1033 feet per second.

Velocity in solids and liquids.—By striking with a hammer one end of a long series of connected iron pipes and determining the interval between the sound transmitted by the pipes and that conveyed by the air, the relative velocity of sound in iron and air was determined by two French investigators. The total length of the pipes was 951 metres and the temperature of the air was 11° C. Calculations showed that at this temperature the sound waves in air would take 2·8 seconds to travel 951 metres. The sound was heard through the iron 2·5 seconds before that transmitted by the air, so that it took only 0·3 second to travel along the pipes. The sound was therefore transmitted by the iron about nine times (2·8/0·3) faster than by the air.

The velocity of sound in water was determined by two observers on the Lake of Geneva in 1826. Two boats were moored about eight miles apart. Suspended from one was a large bell immersed in the water and from the other a tube with a trumpet-shaped receiver to catch the sound. When the bell was struck some gunpowder was ignited, and the interval between seeing this light and hearing the sound of the bell gave the velocity of sound in the water. The value found was about 1430 metres per second. These direct methods of determining the velocity of sound are chiefly of historical interest; indirect methods are now employed in most cases. Thus, the formula $V = \sqrt{E/D}$ can be applied to solids or liquids, provided that the elastic constant E is known for the direction in which the waves travel, and is expressed in suitable units. Methods of determining the velocity in bodies which can be obtained in the form of rods are described in Chapter XXVIII.

EXERCISES ON CHAPTER XXVI.

1. Explain how the velocity of sound in air may be measured by two observers stationed some distance apart. How could the measurement be made independent of the velocity of the wind between the stations?

2. A man stationed between two parallel cliffs fires a gun. He hears the first echo after two seconds, and the next after five seconds. What is his position between the cliffs, and when will he hear the third echo?

3. A rifle-bullet is fired against a target one mile distant with an average velocity of 1200 ft. per second. Does the bullet, or the sound of the firing, reach the target first? If the temperature of the air is 61° F., what is the interval of time between the two arrivals?

4. Describe a method of determining the velocity of sound in air. Will the result obtained be the same in summer and in winter? Give reasons for your answer.

5. Two sources of sound A and B are situated at distances of 100 metres and 300 metres respectively from an observer, who estimates the intensity of the sound from A to be four times as great as that from B. Compare the amplitudes of vibration of the two sound waves (i) near the observer, (ii) at equal but small distances from the respective sources.

6. Describe a method of determining the velocity of sound in the open air. How will the result be affected by wind, and how can the effect of wind be allowed for?

7. How is sound propagated? Is the velocity of sound in air constant?

8. If the velocity of sound in a gas varies directly as the square root of the pressure and inversely as the square root of the density of the gas, show what effect change of temperature has on the velocity.

9. How is the velocity of sound in air affected by changes of temperature? Is it affected by changes of pressure?

CHAPTER XXVII.

MUSICAL SOUNDS.

Distinction between a musical note and a noise.—In considering continuous sounds, as distinct from short sharp sounds such as an explosion, it is necessary to realise the physical difference between a **musical note** and a **noise.**

In the case of a musical note, the vibrations are comparatively simple and they fall upon the ear with uniform frequency; and in the case of a noise, the vibrations are complex and of irregular frequency. The sound obtained from an organ pipe and the screech of a parrot are examples of these two classes of sound.

Loudness and pitch of musical notes.—The loudness of a note depends simply upon the amplitude of vibration of the source of the sound: the greater the amplitude the louder the sound.

Observation of a vibrating tuning-fork or of a stretched string will show that as the amplitude of vibration diminishes the loudness of the note diminishes; but the pitch, which depends solely upon the frequency of vibration, remains absolutely the same.

EXPT. 202.—**Smoked-glass record of vibration.** Make a thin metal style, about 1 cm. long, from thin sheet brass, or from brass wire beaten out flat with a hammer, and fix it with wax to one prong of a tuning-fork. Blacken the surface of a glass plate by holding it over the flame of burning camphor or over a yellow gas flame. Lay the glass on a table, strike the tuning-fork, and rapidly draw it across the plate

FIG. 229.—Trace of a vibrating tuning-fork.

so that the smoked surface is just touched by the style. Notice how the amplitude of the wave-trace is greater at the beginning than at the end.

Pitch of a note.—The influence of frequency of vibration on the pitch of a note may be observed by means of **Savart's toothed wheel** (Fig. 230), which consists of a toothed wheel (A) capable of being rotated rapidly while a piece of thin board is held in a position so that the teeth strike against the edge of the board. Although the sound emitted is harsh and unmusical, yet a definite pitch is unmistakable; and, by varying the speed of rotation, it can be made evident that the pitch is raised when the frequency with which the teeth strike the board is increased.

The **disc siren** affords more complete information on the relationship of pitch to frequency of vibration. It consists of a cardboard disc, pierced with several concentric rows of holes, which can be rotated rapidly on a whirling-table.

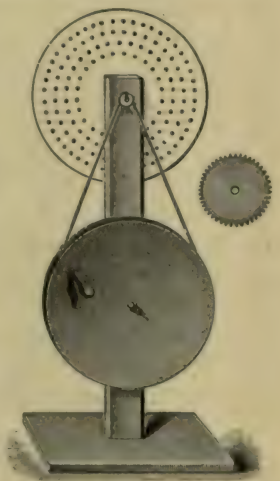

FIG. 230.—Disc siren. The wheel A can be fixed upon the spindle instead of the disc.

The rows, commencing with the innermost, should have 24, 30, 36, and 48 holes respectively. When a stream of air, blown through a narrow jet such as can be made from glass-tubing, is made to impinge on either row of holes of the rotating disc a note of definite pitch is obtained. The sound arises from the fact that when a hole passes in front of the jet a momentary wave of compression is formed immediately behind the disc; and, during the interval between the passage of two consecutive holes, the inertia of the air causes a partial rarefaction to be set up behind the disc. This is followed by a compression when the next hole passes in front of the jet. Thus, we obtain a regular sequence of pulses of compression and rarefaction, which travel outwards with the velocity of sound.

If the speed be increased the same row of holes will give rise to a note of higher pitch.

If the speed remains constant, and the jet be brought successively in front of each row of holes, commencing with the inner row, the series of notes obtained will be recognised by those acquainted with musical intervals as the **third**, **fifth**, and **octave** above the lowest note.

Relation of pitch to rapidity of vibration.—If while the disc siren, described in the previous paragraph, is rotating, the jet be held near to the inner row and then to the outer row, the frequency of the impulses will be in the ratio 24 : 48, or 1 : 2, and the note of higher pitch is an octave above the lower. When the speed of rotation is increased, both notes become sharper, *but the higher note is still an octave above the lower.* When any two musical notes having this striking relationship are sounded, we can always state that the pitch, or rate of vibration, of one note is double that of the other.

Both Savart's wheel and some forms of siren often have an arrangement by which the number of teeth or holes passing a point in one second is recorded. The number of vibrations which produces a note of a particular pitch can thus be determined. Similarly, to find the vibration number of a tuning-fork, or any other body sounding a single note, a Savart's wheel or a siren having a counter connected with it is brought in unison with the note and the number of vibrations per second indicated by the dial is then observed.

A fact of everyday experience will help the student to understand how the pitch of a note depends upon the number of impulses which reach the ear per second. If the whistle of a passing express train be listened to it will be noticed that the pitch of the note while the train is approaching is distinctly higher than when it is receding. The whistle itself is sending out sound waves at exactly equal intervals of time; but, in the interval between the sending of any two consecutive waves, the train will have moved slightly forward, and the distance between any two consecutive condensations (or rarefactions) will be *less* than it would be if the train were stationary. The waves, therefore, are of shorter length, and the pitch of the note correspondingly higher. Similarly, when the train is receding, the wave-length will be correspondingly lengthened, and the pitch lowered. The explanation of this phenomenon is known as **Doppler's principle.**

Musical intervals, and the major diatonic scale.—The ratio between the rates of vibration of two notes is termed the **interval** between the notes. Thus, when the disc siren is rotating at constant speed, the interval between the notes derived from the innermost row of holes and the second row is 30/24, or 5/4. This interval is termed a **major third**. The interval between the notes given by the 2nd and 3rd rows is 36/30, or 6/5: this interval is termed a **minor third**. The interval between the notes of the 1st and 3rd rows is 3/2, and is termed a **major fifth**. Since $3/2 = 5/4 \times 6/5$, it is evident that when two intervals are added together the resultant interval is expressed by their numerical product. If the three notes given by the inner rows of holes in the disc siren, or the three notes C, E, and G, of a piano, are sounded together they form a pleasing combination: it is now known as the **major chord**. The relative frequencies of these notes may be expressed by the numbers 24, 30, and 36; and these numbers have the ratio 4 : 5 : 6.

The major diatonic scale, such as is represented by the sequence of white notes of a piano, commencing with middle C, is built up in the following manner: A second major chord is obtained by starting from C′, the octave of C, and descending in the ratio 6 : 5 : 4. This gives frequencies of 48, 40, and 32; and these correspond to the notes C′, A, and F. This set of three notes is known as the **sub-dominant chord**. Finally, a third major chord is obtained by starting from G, and ascending in the ratio 4 : 5 : 6. This gives frequencies of 36, 45, and 54; and these correspond to the notes G, B, and D′. This is known as the **dominant chord**. The note D′ is above the octave of C, and its lower octave D, having a frequency 27, falls between C and E. Thus we obtain the following sequence of notes into which the octave may be divided:

Notes	C	D	E	F	G	A	B	C′
Vibrations per second	256	288	320	341·3	384	426·6	480	512
Relative frequency	24	27	30	32	36	40	45	48
Interval (compared with C)	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2

The sonometer.—The vibrations of wires or strings can be studied conveniently by means of the **sonometer** or **monochord**,

one form* of which is shown in Fig. 231. The essential parts are a sounding-box with wires stretched along it, upon one of which weights can be hung. Near the ends are fixed metal edges called 'bridges'; and similar bridges, which are movable, are required for the purpose of shortening the length of the vibrating string.

FIG. 231.—A sonometer.

When a string is vibrating the point which has greatest amplitude is termed an **antinode** or **loop**, and the points which are stationary are termed **nodes**. This simplest mode of vibration of a string occurs when there is one antinode, in the centre, and a node at each end. When vibrating in this manner the wire gives out its **fundamental note.**

It will be explained in a subsequent paragraph how, by fixing points of the wire other than the extreme ends, the wire may be made to vibrate in 2, 3, 4, or more separate portions (Fig. 233); in such cases the number of nodes and antinodes is greater than when the wire is giving its fundamental note.

Laws of vibrating strings.—The rate of vibration of a stretched wire or string depends upon the length, the stretching force, and the mass of unit length of the wire. The relationship between the rate and these several conditions is expressed in the equation

$$n = \frac{1}{2l}\sqrt{\frac{F}{m}},$$

where n is the rate of vibration of a string of length l cm., and of mass m gm. per unit length, when stretched by a force F dynes. The above equation suggests that the rate of vibration is

(i) directly proportional to the square root of the stretching force, and

* Experiments are more satisfactory if the sonometer is suspended against a wall in a vertical position, and with its lower end projecting slightly forwards so as to ensure good contact between the wire to which weights are attached and the lower bridge.

(ii) inversely proportional to the length and to the square root of the mass of unit length.

EXPT. 203 (i).—**Length of the wire.** Tune the two wires until they are in unison. Shorten by means of a movable bridge one of the wires A until it gives the octave above its original note as compared with the unaltered wire B. Its rate of vibration is twice its previous rate. Measure its length and note whether this is equal to one-half its previous length.

(ii) By means of another movable bridge tune the previously unaltered wire B until in unison with the shortened wire A. Move the bridge under A until its shorter position gives a note one octave above B, and therefore two octaves above its fundamental note. Its rate of vibration is now four times as great as its initial rate. Measure its length and note whether this is equal to one-fourth of its initial length.

(iii) If two tuning-forks of known rate of vibration are available, measure the lengths of one of the wires required to give notes in unison with the two forks respectively, keeping the tension the same. Note whether the ratio of the lengths is equal to the inverse ratio of the rates of the two forks.

EXPT. 204.—**The stretching force.** Stretch a thin wire on the sonometer with a weight of one kilogram, and tune the other wire to unison. Increase the stretching force to four kilograms, and find by comparison with the other wire whether the note now given is one octave above the previous note.

Try to verify the law when the stretching forces are weights of two and of three kilograms.

EXPT. 205.—**Diameter and nature of the wire.** Select two wires (A and B) of different material, *e.g.* brass and steel, or two wires of the same material but of different diameter. Stretch one of them (A) with a known weight, and determine the length l_1 of the fixed wire C which is in unison with it. Make file marks on the wire A where it touches the bridges, unhang the weight, and cut the wire at the file marks by means of wire cutters. Weigh this length of wire, and determine the mass m_1 of unit length. Stretch wire B with the same weight as before, and determine the length l_2 of the wire C which is in unison with it. Proceed, as before, to find the mass m_2 of unit length of wire B.

If n_1 and n_2 are the frequency of vibration of the wires A and B, then $n_1/n_2 = l_2/l_1$. Also, by the above equation, $n_1/n_2 = \sqrt{m_2/m_1}$. Cal-

culate the values of the ratios l_2/l_1 and $\sqrt{m_2/m_1}$, and observe whether they are equal.

Beats.—When two notes very nearly in unison, and of the same quality, are sounded together, the ear is unable to hear either of them separately, as would be the case when their pitch differs considerably; but the ear can detect a throbbing effect, as though the note were being sounded alternately loudly and softly. This is due to the sequence of waves which originate from the two sources of sound alternately reinforcing and interfering with each other.

Fig. 232 represents two such sets of waves, one represented by a continuous line and the other by a dotted line: the wave-length of the former being slightly longer than that of the latter, but the amplitude is the same in both sets. At A the condensations of

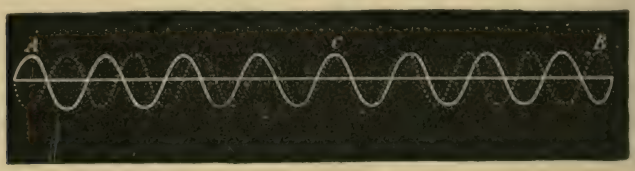

FIG. 232.—Formation of beats.

one set coincide with rarefactions of the other, and the resultant effect is silence. At C the condensations coincide, and the sound will be a maximum. At a further point B the waves again mutually interfere. If the ear be placed at B the momentary silence will be followed by maximum sound when the waves at C reach it, and this will be followed by silence when the waves at A reach it. These pulses of loudness are called **beats.**

It necessarily follows that if two forks vibrate 256 and 257 times respectively in one second their mutual interference will produce one beat in each second. **The number of beats in each second is equal to the difference in the number of vibrations per second made by the two vibrating bodies.** The difference in the number of vibrations must be small for beats to be heard. When more than 16 beats per second are formed, the ear cannot resolve them and a resultant note is heard.

EXPT. 206.—**Beats caused by vibrating wires.** Tune the two wires of a sonometer to apparent unison. Try to detect the presence of

beats: it is easier to detect them when the ear is placed in contact with the end of a wooden rod, the other end of which is pressed against the board of the sonometer. If they cannot be detected, alter *very slightly* the length of one of the wires by means of a movable bridge. Notice how the frequency of the beats increases as the previous unison is more and more disturbed.

EXPT. 207.—**Beats between vibrating wire and tuning-fork.** Adjust the length of a sonometer wire so as to be in perfect unison with a given tuning-fork, and verify the absence of beats by bringing the stem of the vibrating fork in contact with the board of the sonometer. Now load one prong of the fork with a small pellet of wax: this will have the effect of lowering the rate of vibration of the fork, and it should be possible to detect beats when the fork is sounded with the sonometer wire. Attach a larger pellet of wax and observe how the beats are still more frequent.

Harmonics, or overtones.—In previous paragraphs a string has been considered to vibrate with nodes at each end only (Fig. 233 (i)), in which case its fundamental note is produced. But, by preventing the movement of the wire at any intermediate point, a node is set up at that point and the wire vibrates in two or more short segments. Thus, when the wire is touched at its centre (Fig. 233 (ii)) and bowed or plucked at a place halfway between this point and either end, the wire vibrates in two segments. The wire now has two antinodes, A_1 and A_2, or points of maximum movement. When the wire is touched at a point one-third of its length from one end, and then bowed at the middle of the shorter portion, it will divide into three segments (Fig. 233 (iii)). Fig. 233 (iv) shows how the string divides into four segments when touched at a point one-fourth of its length from one end. The positions of these nodes

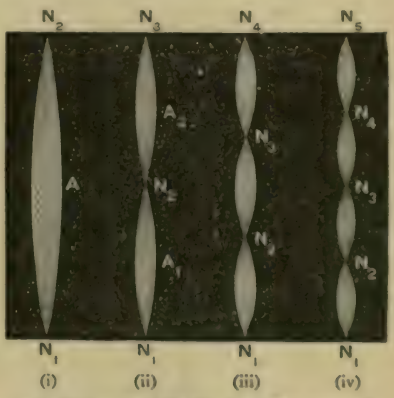

FIG. 233.—Nodes and antinodes of a vibrating string.

and antinodes can be verified by placing upon the wire short narrow strips of paper, usually called riders; those at the nodes remain in their places when the wire is vibrating, but the riders at the antinodes are thrown off.

The pitch of the note emitted depends upon the length of each vibrating segment only; and since, when vibrating in the modes shown in Fig. 233 (i)–(iv), the lengths of the segments have the ratios $1 : \frac{1}{2} : \frac{1}{3} : \frac{1}{4}$, the pitches of the notes emitted have the ratios 1 : 2 : 3 : 4. The lowest note is the **fundamental**; and those produced by the vibration of aliquot parts of the sounding body are termed **harmonics** or **overtones**. In the case of a string vibrating as shown in Fig. 233 (i), the fundamental note only is sounded and is said to be pure. This simple condition, however, is rare; for the vibration of the string may be a combination of the motions shown in both (i) and (ii). When this is the case, the note is not pure, but it is richer on account of the presence of the first harmonic, which is an octave above the fundamental note.

The accompanying diagram (Fig. 234) represents the positions on the musical stave of the sequence of harmonics, which may be

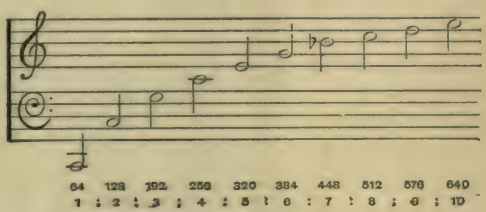

FIG. 234.—Sequence of harmonics.

obtained from a wire tuned in unison with the C note below the bass clef. The **quality** or **timbre** of a note is determined by the presence of these harmonics. Thus, the same note may be sung by the human voice, or played upon a violin, cornet, organ, or other instrument; but though the pitch and intensity may be alike, the timbre differs because of the different number or relative strengths of the harmonics produced when the note is sounded.

EXERCISES ON CHAPTER XXVII.

1. A stretched string 4 feet long is in unison with a tuning-fork which vibrates 256 times a second. What will be the rate of vibration of the string when it has been shortened 6 inches?

2. A sonometer string is stretched with a force of 16 lb. weight. What load must be attached so that the note may be an octave lower?

3. At what point must the G string of a violin be pressed by the finger of the player in order to give the note C'?

4. A fork A has a frequency of 256. When this fork and a second fork B are sounded close together, 3 beats per second can be heard. A pellet of wax is attached to one of the prongs of B, and the frequency of the beats is found to be reduced to 2 per second. What is the frequency of the fork B when unloaded?

5. Determine the vibration number for each tone of a scale the key-note of which has a vibration number 260.

6. A copper wire (density 8·8 gm. per c.c.) 100 cm. long and 1·8 mm. in diameter is stretched by a weight of 20 kilograms. Calculate the number of vibrations which it makes per second when sounding its fundamental note.

7. A given note is sounded first on a piano and then on a violin. How is it that the notes can be distinguished easily though we say the same note has been sounded?

8. How does the frequency of vibration of a stretched string depend upon the length of the string, the stretching force, and any other physical property of the string? How are these laws applied in the piano?

9. If the frequency of a tuning-fork be 128, and the number of vibrations per hour of a second fork exceeds that of the first by 300, how many beats will there be in a minute if the two are sounded together?

10. Describe experiments to show that the impression of a musical interval as judged by the ear depends solely upon the ratio of the frequencies of vibration of the two notes concerned and not upon the difference of their frequencies. The frequencies of vibration of two notes being 400 and 900, what is the frequency of a note that would appear to the ear to lie midway between them?

11. What is the pitch of a tuning-fork? What may be heard when two forks of nearly the same pitch are sounded together? How would you determine which of the two was vibrating the faster?

12. Describe an experiment for showing that, when a musical note is produced, the greater the frequency of the vibrations the higher is the note.

13. Describe some form of siren, and explain how you would use it to determine the frequency of a given tuning-fork.

14. How does the frequency of the note sounded by a string vibrating transversely depend on (i) the length, (ii) the tension of the string?

15. Describe a monochord and explain how to use it to compare the frequencies of two tuning-forks.

If the frequency of the middle C on a pianoforte be 256 vibrations per second, what will be the frequency of the next higher E?

16. A man standing by a railway notices that the pitch of the note due to the whistle of an engine diminishes as the engine passes him. Explain this result.

If the frequency of the whistle is 256 vibrations per second and the velocity of the engine is $\frac{1}{20}$ of that of sound, what will be the frequencies of the notes heard by the man before and after the engine passes him?

17. Explain the formation of the beats heard when two tuning-forks, which are not quite in unison, are sounded together.

A standard fork A has a frequency of 256 complete vibrations per second and, when a fork B is sounded with A, there are 4 beats per second. What further observation is required for determining the frequency of B?

18. A note on a harmonium and a string of a violin have been tuned to be in unison with a given tuning-fork. How do you account for the difference in the quality of the sounds produced by the two instruments?

19. Draw diagrams to show how a stretched wire may vibrate. Upon what does the note emitted by a stretched wire depend?

20. The vibration frequency of a tuning-fork A is 256 complete vibrations per second, and the frequency of another fork X differs from that of A by 4 complete vibrations per second. Describe what will be heard when the two forks are sounded simultaneously, and explain how to determine whether the frequency of X is greater or less than that of A.

21. Explain the meaning of the 'pitch' and the 'intensity' of a musical sound.

How do they respectively depend on the nature of the sound wave which produces the note?

22. A heavy goods train was approaching a railway station. To an observer at the station the puffs of steam from the funnel appeared at a certain instant to coincide with the sound of the blasts. Presently, however, the sounds seem to precede the puffs, the difference between them continually increasing until a second coincidence was established and the process was repeated. Explain this.

CHAPTER XXVIII.

INDUCED VIBRATIONS.

Natural and impressed periods of vibration.—In order to understand clearly the phenomenon of **resonance** it is necessary to distinguish **free** vibrations from **forced** vibrations.

Every simple pendulum has a natural period of vibration when swinging freely—the period depending upon the length of the pendulum. But, by taking hold of the bob with the hand it is possible to impart to it any rate of vibration we please; in this case a forced vibration is produced.

The sound of a tuning-fork can be heard when the fork is held close to the ear. But if the handle of the vibrating fork be brought into contact with a board or table the sound can be heard at a considerable distance. The reason for this is that the vibrations are transmitted through the handle to the board, which is thus forced to vibrate at the same rate. The waves set up in the air by the vibrating board are added to those originating from the fork, and the sound seems much louder. The board is thus caused to vibrate at a rate which is not necessarily its natural rate; or, in other words, it is in a state of **forced vibration.** The tone of a violin is due, to a considerable extent, to the forced vibrations set up in the wooden case of the instrument; similarly, the tone of a piano is due to the forced vibrations set up in the sounding-board across which the wires are stretched.

Induced vibrations.—Free vibrations may be set up in a heavy pendulum, or in any other suspended body, by applying a sequence of small repeated blows, providing that the interval between the blows corresponds to the natural period of free

vibration of the suspended body. If, however, the blows come irregularly or at an incorrect frequency, they may have little or no effect in setting up vibration. A regiment of soldiers crossing a suspension bridge may set up dangerous oscillations if the frequency of their step corresponds with the natural period of vibration of the bridge: for this reason the soldiers are often ordered to fall out of step when crossing such a bridge. The same effect may be observed when walking along a plank bridge; for considerable oscillation may be set up if the steps are rightly timed, but the oscillations cease when the rate of step is increased or diminished.

EXPT. 208.—**Sympathetic vibrations.** (i) Select two tuning-forks of the same pitch, one of them being fixed upon a sounding-box or resonator as in Fig. 235. Strike the other fork and hold its stem upon the sounding-box for a moment; then remove it and stop its vibration. Notice that the fork upon the sounding-box has taken up the vibrations, and gives out the same note.

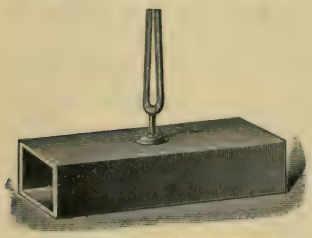

FIG. 235.—Tuning-fork on a resonator.

(ii) Stretch a wire upon the sonometer until it is in unison with a tuning-fork. Set the fork in vibration and hold the stem for a moment upon the sonometer. Notice that the wire has taken up the vibrations and sounds the same note.

(iii) Sing any note loudly near a piano and stop suddenly. The note in unison with it will be sounded by the piano.

It is easy to realise the process by which such waves of sound set up vibrations in a wire in unison with the note sounded. For, suppose a condensation to strike the wire; this will thrust the wire slightly aside in the direction in which the waves are travelling. During the passage of the succeeding rarefaction, the wire has time to swing back past its position of rest, when it is thrust forward again by the next condensation. Thus, a series of slight, but well-timed, impulses are given to the wire, which soon acquires a considerable amplitude of vibration. By such reasoning it is clear that a wire not in unison with the note will not be affected so readily by the waves of sound.

The familiar phenomenon of the sound obtained by blowing across the open end of a key shows that vibrations may be set up

in an air column; and an air column of definite length has a definite natural period of vibration. When a vibrating tuning-fork is held over a tall glass cylinder, into which water is poured gradually so as to vary the length of the air column, a length can be obtained which will resound loudly to the note of the tuning-fork.

The term **Resonance** is applied to all such effects of vibratory motion produced in one body by the influence of another.

Vibrations of air columns.—The circumstances in which a vibratory condition may be set up in an air column which is closed at one end may be compared to those which determine a vibratory condition in a spiral spring of which one end is fixed. Thus, suppose a weight to be suspended from a spiral spring (Fig. 236 (ii)); by imparting a succession of small taps, directed upwards, to the weight, a considerable vibration may be set up in the spring, *providing that the frequency of the taps coincides with the natural period of vibration of the spring.* In the same manner, when a succession of small taps is imparted to the open end of an air column (Fig. 236 (i)) by means of a vibrating tuning-fork, considerable vibratory motion is set up in the air column if the rate of vibration of the fork coincides with the natural period of vibration of the air column; and the sound waves originating from the fork are augmented considerably by those from the vibrating air column. It is important to notice, in this analogy, that the fixed end of the spring and the closed end of the air column are stationary, and that the opposite ends in each case are regions of maximum motion.

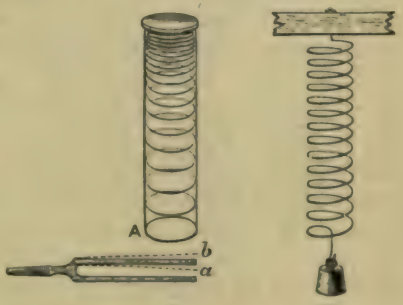

FIG. 236.—Vibrations of a spring and a closed air column.

Tube closed at one end.—A **stationary** vibration is the only type which can be set up within a column of air contained in

a cylinder closed at one end. The relation between the length of the air column and the wave-length of the note given out when the air column is vibrating in its simplest manner may be derived by comparing it with a stretched string in a state of stationary vibration. In the case of the string there are points of maximum motion called antinodes, mid-way between the fixed points called nodes. In the case of the air column, the air near the closed end must be analogous to the node of the vibrating string, since the closed end prevents motion; but, at that end, rapid alternations of condensation and rarefaction are possible. The conditions at the open end are just the reverse, for the air in that position may be in a state of rapid movement; but, since it is exposed freely to the outer air, changes in density are impossible. In other words, *the closed end will be a region of maximum changes of density, and the open end will be a region of maximum motion.* The analogy between a vibrating string and a vibrating air column is shown in Fig. 237. In the former the distance between a node N and an antinode A is equal to *one-fourth* of a wave-length; similarly, the length of the air column B is equal to *one-fourth* of the wave-length of the note which it will emit. This can be verified by the following experiment.

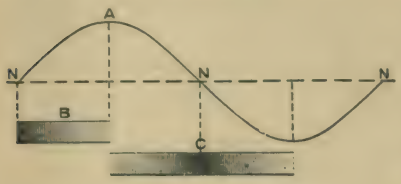

FIG. 237.—Relation between wave-length and vibrating columns of air.

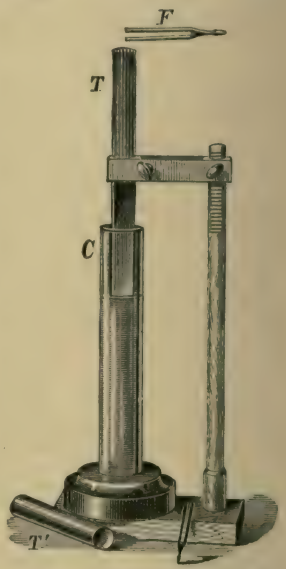

FIG. 238.—Determination of the velocity of sound in air by means of a vibrating air column.

EXPT. 209.—**Resonance of air column.** Support a glass tube T (Fig. 238), about 20 cm. by 3 cm., open at both ends, in a vertical

position with its lower end dipping into water contained in a wider cylinder C. Hold over the upper end of tube T a vibrating tuning-fork F, of which the rate of vibration is known; and adjust the position of T so that the greatest reinforcement of the sound is obtained. Measure the distance from the top of T to the water level. Put T out of adjustment, and repeat the observation at least four times, and take the mean of these results.*

If the length of the air column, as measured above, be l, the wave length of the note emitted is $4l$; and if n be the frequency of vibration of the fork, the velocity v of sound in air, at the temperature of the room, is given by the equation $v = n \times 4l$. Calculate the value of v.

Note the temperature of the room, and calculate the theoretical value from the formula $v = (33200 + 60t)$ cm.; where t is the temperature measured on the Centigrade scale.

By similar experiments the velocity of sound in any gas may be determined. When the column of gas in a closed tube responds to a certain note, the wave-length of the note is four times the length of the column. Suppose the vibration-frequency of the note to be known by comparison with the note of a tuning-fork or other standard, then the velocity of sound in the gas is this number multiplied by four times the length of the vibrating column. Similarly, to find the vibration number of a tuning-fork, a column of air is adjusted as in Expt. 209 until it is in sympathetic vibration with the fork, and its length is measured. Let this be 0·22 metre. Then taking the velocity of sound to be 340 metres per second at the temperature of the air column, we have.

$$V = nl.$$

But $$l = 4 \times 0{\cdot}22 = 0{\cdot}88.$$

Hence $$n = 340/0{\cdot}88$$
$$= 386.$$

Tube open at both ends.—In the case of an air column enclosed in a tube open at both ends, the ends of the tube are always antinodes; and, when the fundamental note is sounded there is a node at the middle of the tube. By analogy with stationary vibrations in a string, as shown in Fig. 237, C, the wave-length of the fundamental note is equal to twice the length of the tube. This can be verified by holding the same fork which was used in Expt. 209 in front of an open tube, the effective length

* Strictly speaking, the position of the antinode is slightly *outside* the end of the tube; and this distance depends on the diameter of the tube. A more exact measure of the quarter wave-length is obtained by adding 0·8 of the radius of the tube to the length measured from the water surface to the top of the tube.

of which can be adjusted by means of a paper cylinder made to slide over the outside of the tube. The length which gives maximum reinforcement will be found to be twice as long as the air column closed at one end.

Organ pipes.—The conditions of vibration of air columns in closed or open tubes apply to organ pipes. In the case of reed pipes, a small tongue is set in vibration at the mouthpiece by air being forced past it; but in ordinary organ pipes, the air blasts strike against a lip, as with a boy's whistle-pipe, and vibrations are thus set up which are reinforced by sympathetic vibrations of the air column. Vibrations of many wave lengths are produced at the mouthpiece, but only those with which the air column is in unison are taken up and strengthened to form the musical note. As described for tubes, the wave-length of the fundamental note given by an organ pipe closed at one end is four times the length of the pipe, but if the pipe be open at both ends, the wave-length of the fundamental note is twice the length of the pipe. The wave-length of the note of an organ pipe can thus be determined from the length of the pipe. Neither the material of which the pipe is made, nor the diameter of the pipe, provided it is small in comparison with the length, need here be considered to affect the pitch of the note produced. The pitch of the note varies directly, however, with the velocity of sound in the vibrating column, and therefore rises as the temperature increases or when a lighter gas than air fills the pipe. As the pitch of a note depends upon the frequency or number of vibrations per second, the equation $V = nl$ (p. 33c) can be used to determine the velocity of sound in a gas when the vibration frequency of the note is known and the length of the pipe emitting it. The following examples illustrate the use of this relation.

FIG. 239.—Organ pipes.

EXAMPLE 1. An open organ pipe 0·65 metre long gives the note middle C, the vibration number of which is 256 per second. Find the velocity of sound in the air at the temperature of the tube.

The wave-length (l) of the note is twice 0·65 metre, that is, 1·30 metres. So that

$$V = 256 \times 1{\cdot}30$$
$$= 332{\cdot}8 \text{ metres per second.}$$

EXAMPLE 2. The velocity of sound in hydrogen is 1269·5 metres per second. What would be the length of a closed organ pipe which gives a note having a vibration frequency of 512 per second when blown with hydrogen?

$$1269{\cdot}5 = 512 \times l.$$

Therefore
$$l = 1269{\cdot}5/512$$
$$= 2{\cdot}48.$$

The length of a closed organ pipe is $l/4$; hence the answer required is $2{\cdot}48/4 = 0{\cdot}62$ metre.

Longitudinal vibrations of rods.—When a rod is clamped at one end and made to vibrate longitudinally, the wave of compression set up travels along the length of the rod in much the same way as it does in a column of air. The relative velocities of sound in the rod and in air can therefore be determined by measuring the length of a closed air column which resounds to the same note as that emitted by a rod clamped at one end and set in vibration. Similarly, the velocity in rods of different material can be found by cutting the rods until they give the same note; for then the relative lengths give the relative velocities.

With a rod clamped at one end and set in longitudinal vibration the point of maximum motion (antinode) is at the free end and that of maximum compression (node) is at the clamped end. The vibration of such a rod may be compared with that of the air in a closed tube or organ pipe. The wave-length of the fundamental note is, therefore, four times the length of the rod. Similarly, the longitudinal vibrations in a rod clamped at the middle are like those of the air in an open tube or organ pipe, for both ends are antinodes. The wave-length of the fundamental note of a rod clamped in this way is twice the length of the rod.

EXPT. 210.—**Longitudinal vibration.** Clamp a rod of brass loosely at its middle. Near one end hang a pellet of sealing-wax. Rub the other end with a resined leather. The rod gives out a note and the pellet is knocked away.

EXPT. 211.—**Pitch and length.** Obtain two long rods of the same kind of wood, one twice the length of the other, and clamp them

separately at the centre. Set the rods in longitudinal vibration by rubbing them with a resined leather. The longer rod will be found to give a note an octave lower than the shorter one, because a wave of compression has to travel twice as far, and consequently appears half as often.

EXPT. 212.—**Relative velocities in deal and oak.** Obtain rods of equal length in deal and oak and set them in longitudinal vibration. The deal rod gives out the higher note because the waves are propagated quicker by it than by the oak. Cut down the length of the oak rod until the two rods give the same note. It will be found that a deal rod 72 in. long emits the same note as an oak rod 49 in. long. The relative velocities of sound in deal and oak are therefore as 72 is to 49.

EXPT. 213.—**Determination of velocities.** Adjust a monochord string until it is in unison with a tuning-fork of known pitch. Measure the length of the string. Set a rod of mahogany clamped at the middle in longitudinal vibration, and adjust the monochord string in unison with it. Measure the length of this vibrating string. Knowing the pitch of the fork, the velocity of sound in mahogany can be found as follows:

Standard fork C′ = 543 vibrations per second.

Length of string in unison with C′ = 60

" " " rod = 24

Length of rod = 6 feet.

Then $\frac{24}{60} = \frac{543}{n}$

Therefore the frequency, $n = 1357{\cdot}5$.

Using the formula $V = nl$,

we have $n = 1357{\cdot}5$

and l = twice the length of the rod.

Hence velocity of sound in mahogany

$= 1357{\cdot}5 \times 6 \times 2$

$= 16290$ feet per second.

Find in the same way the velocity of sound in glass, oak, and brass.

EXERCISES ON CHAPTER XXVIII.

1. A tuning-fork produces strong resonance when held over a jar 22·35 cm. long and 2 cm. radius. Find the wave-length of the note emitted. If the temperature is 15° C., calculate the rate of vibration of the fork.

2. State how the air moves in different parts of a tube 1 ft. long, open at both ends, when sounding its fundamental note. Neglecting

the correction for the width of the tube, and assuming the velocity of sound in air to be 1116 ft. per second, calculate the frequency of the note emitted.

3. Find the number of vibrations per second of a fork which produces resonance in a pipe which is 10 inches long and 2 inches in diameter, and closed at one end. The temperature of the air at the time of the experiment is 50° F.

4. Explain the meaning of the term 'Resonance' by reference to an experiment in which a vibrating tuning-fork is held over a column of air. Show how this experiment can be used to determine the vibration frequency of a fork, the velocity of sound in air being assumed as 1100 ft. per second.

5. If there are 32 holes in the disc of a siren, which makes 1050 revolutions per minute, what is the frequency of the note emitted? What would be the length of an open organ pipe which, sounding its fundamental, emitted the same note? (Velocity of sound in air = 1120 ft. per sec.)

6. The end of one of the prongs of a tuning-fork is held over the mouth of a tube which can be raised or lowered in water. When the mouth of the tube is at a given height above the water the sound of the fork appears to swell out loudly. Carefully explain this. Would the height be different, if (i) the temperature of the air were higher, (ii) if the air in the tube were replaced by carbonic acid gas? Give reasons for your answer.

7. A glass tube one foot long and one inch in diameter is closed at one end. Describe the motion of the air within the tube when vibrating in the simplest possible manner. How, if at all, would the pitch of the note emitted by such a column be changed by a rise of temperature, by an increase of atmospheric pressure, and by the substitution of a denser gas for the air in the tube?

8. How would you determine the number of vibrations per second executed by the prong of a tuning-fork?

9. Explain the effect of temperature upon the frequency of the note emitted by an open organ pipe. Does the effect depend upon the material of which the pipe is made, or upon the nature of the gas in which the pipe is sounded? Give reasons.

10. In building an organ for use in a warm climate it is necessary, in order to produce notes of a given pitch, to make the pipes longer than if they were to be used in England. Explain why this is so.

11. How would you show that the velocity of sound is not the same in air as in carbon dioxide gas at the same temperature?

12. What is meant by resonance?
Give two illustrations of its use in acoustic experiments.

13. Explain what is meant by resonance. The length of the column of air in a tube closed at one end which gives the greatest

resonance with a tuning-fork is observed to be 32·5 cm.; find the wave-length of the note emitted by the fork.

14. Describe the method of determining the velocity of sound in air by means of a resonance tube and a tuning-fork of known pitch.

How may the correction due to the 'open end' of the tube be determined experimentally?

15. You are given a tuning-fork of known frequency, a deep gas jar, and a metre scale. How would you determine the velocity of sound in air?

On what acoustic principle does this experiment depend?

16. State and explain what may be noticed when a person sings a note in front of the strings of a piano.

17. How would you determine the vibration number of a tuning-fork?

18. How would you show that the velocity of sound in air is different from its velocity in carbon dioxide?

Explain how it is possible for a man to calculate roughly his distance from a cliff when the velocity of sound in air is known.

19. Why, when the handle of a vibrating tuning-fork is put in contact with a wooden board, is the amount of sound produced greatly increased? Is the time during which the fork goes on vibrating affected?

20. If you were provided with a fork of unknown pitch and the necessary apparatus, how would you determine the velocity of sound in air?

21. State how you could determine the velocity of sound in a solid which could be obtained in the form of a rod.

PHYSICAL TABLES.*

(4) Density, or Mass per Unit Volume.

COMMON SOLIDS.

SUBSTANCE.	MASS OF 1 CUBIC CENTIMETRE IN GRAMS.	MASS OF 1 CUBIC FOOT IN LBS.
Anthracite	1·4–1·8	87–112
Beeswax	0·96–0·97	60–61
Brick	1·4–2·2	87–137
Butter	0·86–0·87	53–54
Caoutchouc	0·92–0·99	57–62
Chalk	1·0–2·8	118–175
Ebonite	1·15	72
Felspar	2·55–2·75	159–172
Flint	2·63	164
Glass (Common)	2·4–2·8	150–175
„ (Flint)	2·9–5·9	180–370
Granite	2·64–2·76	165–172
Graphite	2·30–2·72	144–170
Ice	0·917	57·2
Limestone	2·68–2·76	167–171
Magnetite	4·9–5·2	306–324
Marble	2·6–2·84	160–177
Paper	0·7–1·15	44–72
Paraffin	0·87–0·91	54–57
Pitch	1·07	67
Porcelain	2·3–2·5	143–156
Pumice Stone	0·37–0·9	23–56
Quartz	2·65	165
Rock Salt	2·18	136
Sand (Silver)	2·63	164
Sandstone	2·14–2·36	134–147
Slate	2·6–3·3	162–205
Starch	1·53	95
Sugar	1·61	100
Tallow	0·91–0·97	57–60

* Chiefly compiled from "Physical Tables," published by the Smithsonian Institution, Washington, U.S.A.

METALS AND ALLOYS.

SUBSTANCE.	MASS OF 1 CUBIC CENTIMETRE IN GRAMS.	MASS OF 1 CUBIC FOOT IN LBS.
Metals: Aluminium	2·56–2·80	160–175
Copper (Cast)	8·30–8·95	517–558
,, (Drawn)	8·93–8·95	556–558
Gold	19·3	1205
Iron (Cast)	7·03–7·73	438–482
,, (Wrought)	7·80–7·90	486–493
,, (Steel)	7·60–7·80	474–486
Lead	11·36	709
Magnesium	1·74	108·5
Mercury (at 0° C.)	13·596	848
Nickel	8·6–8·90	536–555
Platinum	21·37	1331
Silver	10·40–10·57	649–659
Tin	7·30	455
Zinc	7·04–7·19	439–449
Alloys: Brass	8·44–8·70	526–542
Bronze	8·74–8·89	545–555
Coins (British): Gold	17·72	1106
Silver	10·31	643
Bronze	8·96	559
German Silver	8·30 – 8·77	518–547
Gunmetal	8·0–8·4	499–524

LIQUIDS.

SUBSTANCE.	MASS OF 1 CUBIC CENTIMETRE IN GRAMS.	MASS OF 1 CUBIC FOOT IN LBS.
Alcohol (Ethyl)	0·807	50·4
,, (Methyl)	0·810	50·5
,, (Proof Spirit)	0·916	57·2
Benzene	0·899	56·1
Carbolic Acid (Crude)	0·950–0·965	59·2–60·2
Carbon Bisulphide	1·293	80·6
Ether	0·736	45·9
Glycerine	1·260	78·6
Naphtha (Wood)	0·810–0·848	50·5–52·9
Oil, Camphor	0·910	56·8
,, Castor	0·969	60·5
,, Linseed	0·942	58·8
,, Mineral	0·900–0·925	56·2–57·7
,, Olive	0·918	57·3
,, Turpentine	0·873	54·2
Petrol	0·68–0·72	42·4–44·9
Petroleum	0·878	54·8
Sea Water	1·025	64·0
Water	1·000	62·4

GASES.

Substance.	Specific Gravity.	Mass of 1 Litre in Grams.	Mass of 1 Cubic Foot in Lbs.
Air	1·000	1·293	0·0807
Carbon Dioxide	1·529	1·974	0·1232
Hydrogen	0·0696	0·089	0·0056
Nitrogen	0·972	1·250	0·0780
Oxygen	1·105	1·430	0·0893
Steam at 100° C.	0·469	0·581	0·0363

WOODS.

Substance.	Mass of 1 Cubic Centimetre in Grams.	Mass of 1 Cubic Foot in Lbs.
Ash	0·65–0·85	40–53
Beech	0·70–0·90	43–56
Birch	0·51–0·77	32–48
Box	0·95–1·16	59–72
Cedar	0·49–0·57	30–35
Cherry	0·70–0·90	43–56
Cork	0·22–0·26	14–16
Ebony	1·11–1·33	69–83
Elm	0·54–0·60	34–37
Lignum Vitae	1·17–1·33	73–83
Lime	0·32–0·59	20–37
Mahogany (Spanish)	0·85	53
Maple	0·62–0·75	39–47
Oak	0·60–0·90	37–56
Poplar	0·35–0·5	22–31
Sycamore	0·40–0·60	24–37
Teak	0·66–0·98	41–61
Walnut	0·64–0·70	40–43
Willow	0·40–0·60	24–37

(5) Acceleration due to Gravity.

Latitude.	Increase of Velocity per Second due to the Earth's Attraction.		
30°	979·3 cm.	385·5 in.	32·13 ft.
40	980·1	385·9	32·16
50	981·0	386·2	32·19
60	981·9	386·6	32·22

(6) Length of Seconds Pendulum.

Latitude.	Centimetres.	Inches.
30°	99·22	39·06
40	99·31	39·10
50	99·40	39·13
60	99·49	39·17

(7) Pressure.

A standard atmosphere is the pressure of a vertical column of pure mercury, having a height of 760 mm. and temperature 0° C. under standard gravity at latitude 45° and at sea level.

1 standard atmosphere = 1033 grams per sq. cm.
= 14·7 lbs. per sq. in.
= 2116 lbs. per sq. ft.

Pressure of mercurial column 1 inch high
= 34·5 grams per sq. cm.
= 0·491 lbs. per sq. in.

A column of water 2·3 ft. high corresponds to a pressure of 1 lb. per square inch.

(21) Indices of Refraction Relative to Air.

Solids.			
Crown Glass	1·53	Fluor Spar	1·43
Diamond	2·42	Ice	1·31
Emerald	1·58	Rock Salt	1·54
Flint Glass	1·62	Ruby	1·77
Liquids.			
Alcohol	1·36	Olive Oil	1·47
Benzene	1·49	Sulphuric Acid	1·42
Carbon Bisulphide	1·63	Turpentine	1·46
Glycerin	1·47	Water	1·33

(22) Velocity of Sound.

Substance.	Metres per Second.	Feet per Second.
Aluminium	5104	16740
Brass	3500	11480
Copper	3560	11670
Iron	5130	16820
Platinum	2690	8815
Silver	2610	8553
Marble	3810	12500
Slate	4510	14800
Glass	5000–5300	16410–17380
Ivory	3013	9886
Ash, along the fibre	4760	15310
„ across the rings	1390	4570
„ along the rings	1260	4140
Oak	3850	12620
Pine	3320	10900
Poplar	4280	14050
Alcohol	1264	4148
Turpentine	1326	4350
Water	1437	4714
Temp. 0° C. Air	332	1090
Carbon Dioxide	258	846
Ammonia	415	1361
Hydrogen	1286	4221
Illuminating Gas	490	1609
Oxygen	317	1041

(27) Trigonometrical Ratios.

Angle: Deg.	Sine.	Tangent.	Cotangent.	Cosine.	
0°	0	0	∞	1	**90°**
1	0·0175	0·0175	57·2900	0·9998	89
2	0·0349	0·0349	28·6363	0·9994	88
3	0·0523	0·0524	19·0811	0·9986	87
4	0·0698	0·0699	14·3006	0·9976	86
5	0·0872	0·0875	11·4301	0·9962	**85**
6	0·1045	0·1051	9·5144	0·9945	84
7	0·1219	0·1228	8·1443	0·9925	83
8	0·1392	0·1405	7·1154	0·9903	82
9	0·1564	0·1584	6·3138	0·9877	81
10	0·1736	0·1763	5·6713	0·9848	**80**
11	0·1908	0·1944	5·1446	0·9816	79
12	0·2079	0·2126	4·7046	0·9781	78
13	0·2250	0·2309	4·3315	0·9744	77
14	0·2419	0·2493	4·0108	0·9703	76
15	0·2588	0·2679	3·7321	0·9659	**75**
16	0·2756	0·2867	3·4874	0·9613	74
17	0·2924	0·3057	3·2709	0·9563	73
18	0·3090	0·3249	3·0777	0·9511	72
19	0·3256	0·3443	2·9042	0·9455	71
20	0·3420	0·3640	2·7475	0·9397	**70**
21	0·3584	0·3839	2·6051	0·9336	69
22	0·3746	0·4040	2·4751	0·9272	68
23	0·3907	0·4245	2·3559	0·9205	67
24	0·4067	0·4452	2·2460	0·9135	66
25	0·4226	0·4663	2·1445	0·9063	**65**
26	0·4384	0·4877	2·0503	0·8988	64
27	0·4540	0·5095	1·9626	0·8910	63
28	0·4695	0·5317	1·8807	0·8829	62
29	0·4848	0·5543	1·8040	0·8746	61
30	0·5000	0·5774	1·7321	0·8660	**60**
31	0·5150	0·6009	1·6643	0·8572	59
32	0·5299	0·6249	1·6003	0·8480	58
33	0·5446	0·6494	1·5399	0·8387	57
34	0·5592	0·6745	1·4826	0·8290	56
35	0·5736	0·7002	1·4281	0·8192	**55**
36	0·5878	0·7265	1·3764	0·8090	54
37	0·6018	0·7536	1·3270	0·7986	53
38	0·6157	0·7813	1·2799	0·7880	52
39	0·6293	0·8098	1·2349	0·7771	51
40	0·6428	0·8391	1·1918	0·7660	**50**
41	0·6561	0·8693	1·1504	0·7547	49
42	0·6691	0·9004	1·1106	0·7431	48
43	0·6820	0·9325	1·0724	0·7314	47
44	0·6947	0·9657	1·0355	0·7193	46
45	0·7071	1·0000	1·0000	0·7071	**45**
	Cosine.	Cotangent.	Tangent.	Sine.	Angle: Deg.

ANSWERS TO NUMERICAL EXERCISES

Chapter XVIII. (p. 245.)

5. 7·8 ft. 6. $(91{\cdot}7 \times 10^4)$ miles. 8. 78·5 sq. ft.
9. 7·8 c.p. 10. 16 : 9. 11. 2·5 : 1.
12. 42 cm. from the candle; 70 cm. on the distant side of the candle.
13. The 100 c.p. lamp. 14. 2·56%. 15. 75%.

Chapter XIX. (p. 256.)

4. 60°.

Chapter XXI. (p. 287.)

3. 2 ft. from the mirror; relative size $= \frac{1}{3}$.
4. 15 cm. behind the mirror; 1·87 cm. long. 6. Diameter = 0·03 ft.
17. 46·5 cm.; 28 cm.; 26 cm.

Chapter XXII. (p. 301.)

4. − 15 inches. 5. − 13·3 inches. 8. − 7·5 cm.
9. 7·5 cm. in front of lens; 0·5 cm. long.
10. 22·5 cm. from lens; 52 in. × 38 in. 11. 1 ft. or 2 ft. from candle.
13. + 16 cm. 14. + 33·3 cm. 15. 4·5 in. from lens.
17. 10 cm.; 11·1 cm.; + 9 dioptres. 18. − 16 cm. 19. 11 ft.; 11 in.
23. Convex; 6·15 in.; 10·56 ft. square.

Chapter XXIII. (p. 313.)

1. − 2·4 in. 2. Concave; + 5·14 in. 3. 28·8 cm.
4. 120 cm. 8. − 13·8 in. 9. $f = + 6$ in.; 6 inches. 11. 4 times.

Chapter XXV. (p. 335.)

2. 15 ft. ; 2·5 ft. per sec. **5.** 4 times. **8.** 1104 ft. per sec.

Chapter XXVI. (p. 341.)

3. The bullet ; 0·31 sec. **5.** (ii) 4 : 9.

Chapter XXVII. (p. 351.)

1. 292·6. **2.** 4 lb. **3.** $\frac{1}{4}$ of its length from the end. **4.** 259.
6. 46·8. **9.** 5. **10.** 600. **16.** 269·5 and 243·8.

Chapter XXVIII. (p. 360.)

1. 360, approximately. **2.** 558. **3.** 307·8. **5.** 560 ; 6 inches.

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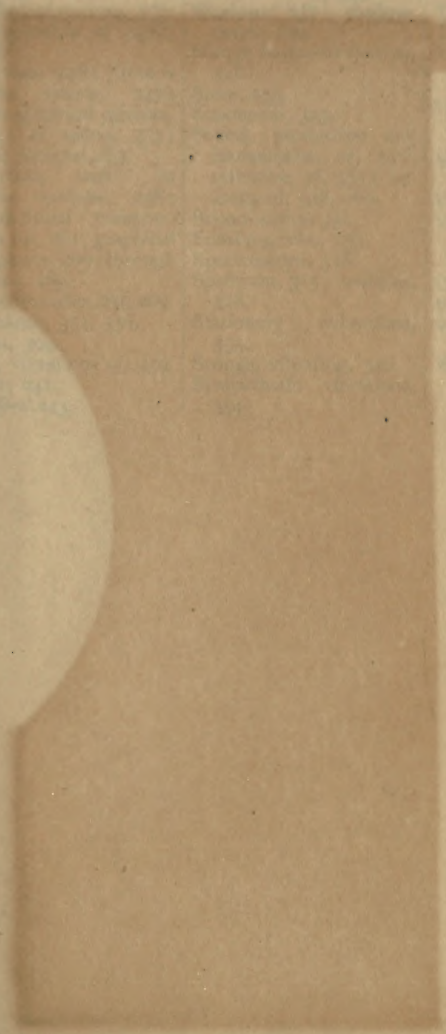

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